Guide to the Framework: contents

Foreword

The mathematics strand of the Key Stage 3 National Strategy builds on the achievements and experience of the National Numeracy Strategy for primary schools as well as improving transition from primary to secondary school. It is also a key part of our strategy to transform standards in the early years of secondary school.

We have already seen from primary colleagues that targets are important to challenge, motivate and raise expectations. It is important that we carry this forward into the early years of secondary education. We recently consulted on targets for 14-year-olds, and we believe that they will help schools in providing a clear focus for the first year of the new strategy.

The *Framework for teaching mathematics: Years 7, 8 and 9* is a vital part of a comprehensive support package available to teachers and headteachers. It is based closely on the Programmes of Study for mathematics in the National Curriculum for 2000. The Framework is proving to be a valuable resource in pilot schools and I am encouraged by the positive feedback we have already received from teachers about it and the other material prepared for the strategy by our team of professional advisers.

We recognise this is an ambitious programme of professional development for secondary teachers, but we are confident that by providing the right support, and with teachers' commitment and enthusiasm, we will make a difference to the achievement of pupils.

The Rt Hon David Blunkett MP
Secretary of State for Education and Employment
March 2001

Introduction

The National Numeracy Strategy was established in 1998 with an ambitious target: that 75% of 11-year-olds achieve at least level 4 in the National Curriculum tests by the year 2002. The Framework for teaching mathematics from Reception to Year 6, a key element of the Numeracy Strategy, has helped teachers to set appropriately high expectations for their pupils and ensure progression through the primary years. Primary pupils are already close to achieving the 2002 target.

The challenge now is to secure and build on the pupils' achievements in Key Stages 1 and 2. The National Strategy for Key Stage 3 aims to address this challenge, drawing on the best practice in secondary schools, experience in the 205 schools that have piloted developments in Key Stage 3, and findings from inspection and research about what helps to raise standards.

The Government intends to set ambitious targets for achievement in the National Curriculum tests for mathematics by 2007, with a milestone target for 2004. To reach this goal will require effective teaching, raised expectations of all pupils, and prompt, effective catch-up support for those who need it. That is why the National Strategy focuses on four important principles:

- **Expectations**
  establishing high expectations for all pupils and setting challenging targets for them to achieve
- **Progression**
  strengthening the transition from Key Stage 2 to Key Stage 3 and ensuring progression in teaching and learning across Key Stage 3
- **Engagement**
  promoting approaches to teaching and learning that engage and motivate pupils and demand their active participation
- **Transformation**
  strengthening teaching and learning through a programme of professional development and practical support

The National Strategy for Key Stage 3

As part of the National Strategy for Key Stage 3 schools are expected to:

- set targets for 14-year-olds in English, mathematics and science;
- provide catch-up classes for all Year 7 pupils who did not previously achieve level 4 in English and mathematics;
- participate in the training programmes and follow them through at school level;
- use the English and mathematics Frameworks, or be able to justify not doing so by reference to what they are doing.

The Framework for teaching mathematics from Reception to Year 6, which has received wide support for its use in Key Stages 1 and 2, is extended in this document to Years 7, 8 and 9. The Government believes that mathematics teachers in Key Stage 3 will find the Framework a valuable tool for reviewing and adjusting their practice, and that many will wish to use it for planning their lessons. There is, after all, no point in teachers re-inventing solutions to problems and challenges that are common to all. However, schools should make a professional judgement about this, once they have studied the Framework, reflected on their training, and reviewed their current practice.

The factors below should influence the degree to which a mathematics department adjusts its current practices and follows this Framework in detail:

- pupils' past, current and expected attainment in mathematics and the extent to which the department is likely to meet its target for raising standards;
- the curriculum leadership in the department and the department's effectiveness in evaluating its strengths and weaknesses in mathematics, and in planning further action to maintain or improve standards;
- the quality of the department's planning and monitoring of its teaching, and the extent to which the teaching objectives and expectations of pupils compare with the level and rigour of those in this Framework;
- the quality of the teaching and assessment of mathematics and how these compare with the criteria identified in this Framework;
- the extent to which the department is staffed by non-specialist teachers of mathematics who require support with their planning and teaching;
- whether the department is already involved in an initiative to raise standards in mathematics in Key Stage 3, through a properly supported programme that incorporates practices similar to those described in this Framework, making it possible to continue its own project development alongside the National Strategy.

The school's senior managers will need to keep all these factors in mind when they are supporting and monitoring work in mathematics.

About this document

The *Framework for teaching mathematics: Years 7, 8 and 9* provides teachers with guidance on meeting the National Curriculum requirements for mathematics. Its purposes are:

- to promote continuity and progression between Key Stage 2 and Key Stage 3 by building on pupils' achievements in primary schools;
- to ensure that pupils entering Key Stage 3 below level 4 are helped to catch up;
- to provide practical support and guidance for teachers and trainee teachers of mathematics in Key Stage 3 on planning and teaching appropriately challenging and engaging work for pupils.

The document contains a great deal of detail which, at a first reading, will need to be studied one section at a time. Although addressed mainly to teachers and trainee teachers who will use it for day-to-day reference, it is also important for:

- heads of secondary-school mathematics departments, lecturers and others such as numeracy consultants who guide teachers or lead INSET activities;
- senior managers in secondary schools and LEA inspectors and advisers who monitor and support mathematics teaching and standards.

As well as providing advice on teaching strategies, inclusion and differentiation, and assessing pupils' progress, the document sets out **yearly teaching programmes** showing how objectives for teaching mathematics can be planned from Year 7 to Year 9.

The teaching programmes meet the requirements of the National Curriculum and cover all aspects of the programmes of study for Key Stage 3. They continue and extend the progression and expectations already established in the Framework for Key Stages 1 and 2, so that they correspond to these levels:

Year 5	revision of level 3, but mainly level 4
Year 6	consolidation of level 4, and start on level 5
Year 7	revision of level 4, but mainly level 5
Year 8	consolidation of level 5, and start on level 6
Year 9	revision of level 5, but mainly level 6; objectives for able pupils at level 7, with some at level 8

The National Curriculum states that the great majority of Key Stage 3 pupils are expected to work within the range of levels 3 to 7, and the Key Stage 3 programmes of study for mathematics correspond to these levels. For the small proportion of 14-year-olds who achieve level 8 in the Key Stage 3 tests, you will need to look at and beyond the extension objectives for Year 9 and draw topics from the higher programmes of study for Key Stage 4.

Some of the objectives in the yearly teaching programmes are more critical than others. These **key objectives** are highlighted in bold type because they are central to pupils' achievements in relation to the National Curriculum level descriptions. The key objectives should be given priority when you are planning work and assessing pupils' progress.

Examples of **planning charts** based on the yearly teaching programmes illustrate how mathematical topics can be grouped in units of work throughout each term and over the year (see pages 48–50). A suggested number of hours of teaching is given for each unit with the assumption that time for assessment and review is built into the units.

After the yearly teaching programmes comes a **supplement of examples** to refer to when you are planning day-to-day lessons. The examples are not intended to be covered as a 'scheme of work', or used as a 'textbook' or for worksheets. Their purpose is to illustrate the level of difficulty of each teaching objective in the teaching programmes through a selection of what pupils should know, understand and be able to do by the end of the school year. They can be considered as examples of assessment activities or of pupil target statements. Over time, you can add to and replace the examples with those that you find most useful and cross-reference them to other resources, including textbooks and information and communication technology (ICT).

At the end of the document there is a **vocabulary checklist**, showing the key mathematical vocabulary used in the teaching programmes and examples. This forms an extension to the booklet *Mathematical vocabulary*, described on the next page.

Related publications

A number of other publications complement the Framework. Those marked '(DfEE)' are available from DfEE Publications, tel: 0845 6022260, and are on the Standards website at http://www.standards.dfee.gov.uk/numeracy. Publications marked '(QCA)' can be obtained from QCA Publications, tel: 01787 884444, fax: 01787 312950.

- *Auditing mathematics in Key Stage 3:* a guide in two parts. Part 1 leads you through the judgements to be made and provides space to record them. Part 2 contains supplementary notes. (DfEE)
- *Making links: guidance for summer schools* and *Units of work for summer numeracy schools:* folders containing guidance and teaching materials to support a summer numeracy school. (DfEE)
- *Springboard 7:* a pack of teaching materials providing a two-term catch-up programme of mathematics for underachieving pupils entering Year 7. (DfEE)
- *Bridging units:* two units, *Fractions, decimals and percentages* and *Introduction to algebra*, to help the transition of pupils from Key Stage 2 to Key Stage 3. Each unit covers several lessons to be taught at the end of the summer in Year 6 and in the early autumn in Year 7. (QCA)
- *Numeracy lessons* and *More numeracy lessons:* two books with descriptions of lessons in each year group from Reception to Year 7. (DfEE)
- *Mathematical challenges for able pupils in Key Stages 1 and 2:* challenging problems to extend able primary pupils, linked to the Framework. (DfEE)
- *Teaching mental calculation strategies* and *Teaching written calculation strategies:* two booklets for teachers in Key Stages 1 and 2 describing approaches to the teaching of mental and written calculations and the use of calculators. (QCA)
- *Mathematical vocabulary:* checklists of the important vocabulary for each year group from Reception to Year 6. An introductory section discusses questioning strategies. The booklet is also suitable for reference by Key Stage 3 teachers who support special educational needs (SEN) or English as an additional language (EAL). (DfEE)
- *Springboard 5:* teaching materials providing an intensive 20-lesson programme of mathematics for underachieving pupils in Year 5. Key Stage 3 teachers who support special educational needs may find the materials useful for reference. (DfEE)
- *Standards in mathematics: exemplification of key learning objectives from Reception to Year 6:* examples of pupils' work illustrating the key learning objectives for each age group from Reception to Year 6. Key Stage 3 teachers who support special educational needs may also find this booklet useful. (QCA)

Raising standards in mathematics

The factors that help to promote higher standards in schools have been documented by Ofsted and in school improvement research.

Where the **leadership, management and planning** of mathematics in secondary schools is concerned, better standards occur when:

- the head of department is well informed, provides strong leadership and sets high expectations for what can be achieved by staff and pupils;
- a desire to secure high standards through effective teaching and learning pervades the whole department;
- there are clear targets for raising standards in mathematics, and a manageable plan for achieving them, with regular evaluation of the plan's progress;
- there is systematic monitoring, led by the head of department, of teachers' planning, teaching and assessment, followed up by discussion and feedback;
- there is sufficient timetabled teaching time for mathematics, with lesson time extended through regular homework and other out-of-class activities;
- a scheme of work for mathematics is based on identified teaching objectives, and ensures high expectations, consistent approaches and good progression;
- staff teaching mathematics have opportunities to observe each other teach, and meet regularly to discuss and develop common understanding of the mathematics curriculum and appropriate teaching methods.

Where **teaching** is concerned, better standards of mathematics occur when:

- lessons have clear objectives and are suitably paced;
- teachers convey to pupils an interest in and enthusiasm for mathematics;
- a high proportion of lesson time is devoted to a combination of demonstration, illustration, instruction and dialogue, suited to the lesson's objectives;
- pupils are involved and their interest maintained through suitably demanding and varied work, including non-routine problems that require them to think for themselves;
- regular oral and mental work develops and secures pupils' recall skills and mental strategies, and their visualisation, thinking and communication skills;
- there is whole-class discussion in which teachers question pupils effectively, give them time to think, expect them to demonstrate and explain their reasoning, and explore reasons for any wrong answers;
- pupils are expected to use correct mathematical terms and notation and to talk about their insights rather than give single-word answers;
- written activities consolidate the teaching and are supported by judicious use of information and communication technology (ICT), textbooks and other resources;
- teachers make explicit for pupils the links between different topics in mathematics and between mathematics and other subjects;
- manageable differentiation is based on work common to all pupils in a class, with targeted support to help those who have difficulties to develop their mathematics.

Where **assessment** is concerned, better standards of mathematics occur when:

- pupils understand and are engaged in the assessment process;
- teachers use pupils' contributions to assess their strengths and difficulties, to set group and individual targets for pupils to achieve and to plan the next stage of work;
- assessments include informal observations, oral questioning and occasional tests or special activities designed to judge progress;
- recording systems give teachers the information that they need to plan and report successfully, but are not too time-consuming to maintain.

The role of the head of department

Heads of department have a crucial role to play in implementing the National Strategy in Key Stage 3. Positive and sustained leadership of the mathematics department will ensure that it operates as effectively as possible. Although some tasks may be delegated, the head of department is generally responsible for:

- inspiring and enthusing mathematics staff and supporting their professional development and training;
- auditing mathematics, setting annual targets for the subject and producing an annual development plan outlining the actions needed to achieve the targets;
- reviewing and updating the mathematics scheme of work and teaching resources;
- directing and coordinating numeracy across the curriculum;
- organising mathematics teaching groups and allocating staff to teach them;
- monitoring and evaluating teachers' planning and teaching of mathematics and the assessment of pupils' work and progress;
- liaising with other departments in the school, including the special needs department;
- keeping senior managers informed about the department's plans and progress.

There are other responsibilities which may be delegated to the head of department by the headteacher, such as liaising with partner schools, keeping parents and governors informed about pupils' progress in mathematics, or ensuring that the department plays a full part in whole-school initiatives to raise standards and strengthen teaching and learning.

There are three key responsibilities for the head of department in the context of the National Strategy. The first is to **lead an audit** of the teaching and learning practices in mathematics in Key Stage 3, using the criteria in the Framework and in the Strategy's audit guide. The purpose of the audit is to identify for the school's senior managers and for the department:

- what, if any, changes are needed to the department's work in order to raise standards and reach its targets for mathematics;
- which teachers could benefit most from the training offered through the Strategy.

An outline action plan is then needed to make sure that the identified changes are tackled in order of priority, at a pace that is manageable for the department with the resources available to it, and in a way that ensures that the changes can be sustained.

The second responsibility is to inspire and enthuse colleagues by offering **curriculum leadership** so that all staff teaching mathematics regularly discuss and develop common understanding of how to teach certain mathematical topics or particular groups of pupils. Such discussions help to develop teamwork and consistent approaches. They can lead to refinements of a scheme of work and the preparation of teaching materials that all teachers of mathematics can use. From time to time the discussions can be extended to other departments so that mathematics staff consider with other teachers:

- how mathematics can support the teaching and learning of other subjects and what other subjects can contribute to mathematics;
- the development of pupils' numeracy skills across the curriculum, including the approach to calculation and the use of calculators;
- the use of ICT resources for mathematical activities in other subjects, particularly graphical calculators, spreadsheets and databases.

The third key responsibility is the **monitoring and support role**. Part of the work of the head of department is to be aware of the quality of teaching among mathematics staff and the teaching styles and methods they are using, and to advise and support teachers accordingly. In a thriving department, the head of department occasionally observes mathematics lessons taught by other members of staff. In turn, they are given an opportunity to observe the head of department teaching and to see each other at work. The head of department also reviews regularly with mathematics staff the written work of pupils in different classes in order to monitor the progress of each class and to check that marking and other assessments are being carried out satisfactorily. Observations of this kind are best when they are followed up with feedback and collective discussion, and can be of particular help to newly qualified and non-specialist teachers of mathematics.

Mathematics at Key Stage 3

Numeracy and mathematics

The definition of numeracy in the *Framework for teaching mathematics from Reception to Year 6* is extended here to take account of pupils' growing appreciation of mathematics and the demands of the Key Stage 3 curriculum.

Numeracy is a proficiency which is developed mainly in mathematics but also in other subjects. It is more than an ability to do basic arithmetic. It involves developing confidence and competence with numbers and measures. It requires understanding of the number system, a repertoire of mathematical techniques, and an inclination and ability to solve quantitative or spatial problems in a range of contexts. Numeracy also demands understanding of the ways in which data are gathered by counting and measuring, and presented in graphs, diagrams, charts and tables.

Poor numeracy skills hold back pupils' progress and can lower their self-esteem. Improving these skills is a whole-school matter. Each department should identify the contribution it makes towards numeracy skills so that pupils become confident at tackling mathematics in any context. By Year 9, pupils should:

- have a sense of the size of a number and where it fits into the number system;
- recall mathematical facts confidently;
- calculate accurately and efficiently, both mentally and with pencil and paper, drawing on a range of calculation strategies;
- use proportional reasoning to simplify and solve problems;
- use calculators and other ICT resources appropriately and effectively to solve mathematical problems, and select from the display the number of figures appropriate to the context of a calculation;
- use simple formulae and substitute numbers in them;
- measure and estimate measurements, choosing suitable units, and reading numbers correctly from a range of meters, dials and scales;
- calculate simple perimeters, areas and volumes, recognising the degree of accuracy that can be achieved;
- understand and use measures of time and speed, and rates such as £ per hour or miles per litre;
- draw plane figures to given specifications and appreciate the concept of scale in geometrical drawings and maps;
- understand the difference between the mean, median and mode and the purpose for which each is used;
- collect data, discrete and continuous, and draw, interpret and predict from graphs, diagrams, charts and tables;
- have some understanding of the measurement of probability and risk;
- explain methods and justify reasoning and conclusions, using correct mathematical terms;
- judge the reasonableness of solutions and check them when necessary;
- give results to a degree of accuracy appropriate to the context.

Number: from Key Stage 2 to Key Stage 3

An ability to calculate mentally lies at the heart of numeracy. As a teacher, whether of mathematics or another subject, you should stress the importance of mental calculation methods and give all pupils regular opportunities to develop the skills involved. The skills include an ability to:

- remember number facts and recall them without hesitation;
- use known facts to figure out new facts: for example, knowing that half of 250 is 125 can be used to work out 250 – 123;
- draw on a repertoire of mental strategies to work out calculations such as 326 – 81, 223 × 4 or 2.5% of £3000, with some thinking time;
- understand and use the relationships between operations to work out answers and check results: for example, 900 ÷ 15 = 60, since 6 × 150 = 900;
- approximate calculations to judge whether or not an answer is about the right size: for example, recognise that ¼ of 57.9 is just under ¼ of 60, or 15;
- solve problems such as: 'How many CDs at £3.99 each can I buy with £25?' or: 'Roughly how long will it take me to go 50 miles at 30 mph?'

An increased emphasis in Key Stages 1 and 2 on mental calculation does not mean that written methods are not taught in the primary years, but the balance between mental and written methods and the progression from one to the other become increasingly important in the later years of Key Stage 2 and in Key Stage 3.

Calculation in Key Stages 1 and 2

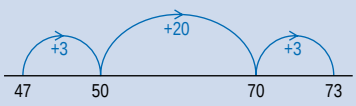
In the early years pupils work orally with numbers. Alongside their oral work they learn to read, interpret and complete statements such as 5 + 8 = ☐ or 13 = ☐ + 5, and then to record the results of their mental calculations in the correct way, using a horizontal format like 43 – 8 = 35.

As they progress to working with larger numbers they learn more sophisticated mental methods and tackle more complex problems. They develop some of these methods intuitively and some they are taught explicitly. Through a process of regular explanation and discussion of their own and other people's methods they begin to acquire a repertoire of mental calculation strategies. It can be hard to hold all the intermediate steps of a calculation in the head and so informal pencil and paper notes, recording some or all of their solution, become part of a mental strategy. These personal jottings may not be easy for someone else to follow but they are an important staging post to getting the right answer and acquiring fluency in mental calculation.

The approach in the primary years builds on the use of the number line, first with numbers marked and then a blank line, to record steps in calculations such as 47 + 26 or 261 – 174.

Jottings

Pupils make jottings to assist their mental calculations, e.g. 47 + 26



They record steps so that they and others can see what they have done, as in:

36 + 27

36 + 20 → 56 56 + 7 → 63

or:

30 + 20 → 50 and 6 + 7 → 13

50 + 13 → 63

Not everyone does a mental calculation like 81 – 26 in the same way (nor is it necessary for them to do so) but some methods are more efficient and reliable than others. Some methods only work for particular cases. Your role is not simply to accept pupils' personal methods but to help them to adopt better ones. By explaining, discussing and comparing different part-written, part-mental methods, you can guide pupils towards choosing and using methods that are quicker, can be applied more widely and are helpful for their future learning.

Standard written methods

Standard written methods are reliable and efficient procedures for calculating which, once grasped, can be used in many different contexts. But they are of no use to someone who applies them inaccurately and who cannot judge whether the answer is reasonable. For each operation, a standard written method should be taught to most pupils, then refined and practised. The method chosen should fulfil several criteria: it should not be too time-consuming to carry out; pupils should be able to explain why it works and apply it reliably; the way it is set out should help them to avoid mistakes; and it should support their further learning of mathematics.

The progression towards written methods is crucial, since standard written methods are based on steps which are done mentally and which need to be secured first. For example, the calculation of 487 + 356, done by the traditional method in columns, requires the mental calculations 7 + 6 = 13, 8 + 5 + 1 = 14 and 4 + 3 + 1 = 8, while a division calculation such as 987 ÷ 23 can involve mental experiment with multiples of 23 before the correct multiple is chosen.

Many countries, and in particular those which are most successful at teaching number, avoid the premature teaching of standard written methods in order not to jeopardise the development of mental calculation strategies. The bridge from recording part-written, part-mental methods to learning standard methods of written calculations begins only when pupils can add or subtract reliably any pair of two-digit numbers in their heads, usually when they are about nine years old.

Standard written methods for addition and subtraction should be well established by Year 6 for nearly all pupils and will be used in Key Stage 3 with an increasing range of whole numbers and decimals. Multiplication and division methods will need to be developed further. When they transfer to Key Stage 3, some pupils may still use informal written methods to record, support and explain their multiplication and division calculations. For example, a few may use a 'grid method' for multiplying two- or three-digit numbers, which you will need to consolidate and build on in Key Stage 3 and relate to work in algebra. For division they may use a 'chunking' method; this too will need to be developed and refined to ensure efficiency. The aim is that where it is appropriate to do so all pupils use standard written methods efficiently and accurately, and with understanding.

When they have reached the stage of working out more complex calculations using written methods, pupils in Key Stage 3 still need to practise and refine their mental calculation strategies. Help them to develop estimation skills in all aspects of calculation, but particularly in multiplication and division. When faced with any calculation, no matter how large or how difficult the numbers may appear to be, encourage them first to ask themselves: 'Can I do this in my head?' They then need to ask: 'Do I know the approximate size of the answer?' and 'Does the answer make sense in the context of the question?' so that they can be reasonably sure that their calculation is right.

Calculator methods

The calculator is a powerful and efficient tool in the right hands. It has an important part to play in subjects such as design and technology, geography, history or science, since it allows pupils to make use of real data from their research or experiments – often numbers with several digits. As with numeracy, the appropriate use of calculators is a whole-school matter. All subjects need to adopt a similar approach and agree when, how and for what purpose calculators are to be used.

Before Year 5, the calculator's main role in mathematics is not as a calculating tool, since pupils are still developing the mental calculation skills and written methods that they will need in later years. But it does offer a unique way of learning about mathematical ideas throughout all key stages. For example, pupils might use a calculator to find two consecutive numbers with a given product and then discuss their methods.

If pupils are to use the basic facilities of a calculator constructively and efficiently for calculating purposes, you will need to teach them in Key Stage 3 the technical skills that they will require. For example, during Key Stage 3 they need to learn:

- how to select from the display the number of figures appropriate to the context of the calculation;
- how to enter numbers and interpret the display when the numbers represent money, metric measurements, units of time or fractions;
- the order in which to use the keys for calculations involving more than one step;
- how to use facilities such as the memory, brackets, the square root and cube root keys, the sign change key, the fraction key, the constant facility, and so on.

By the end of Key Stage 3, pupils should have the knowledge and skills to use a calculator to work out expressions such as

$$3250 \times 1.05^5 \quad \text{or} \quad \sqrt{(7.82^2 - 2.91^2)}$$

All pupils need to continue to learn when it is, and when it is not, appropriate to use a calculator, and their first-line strategy should involve mental calculations. They should have sufficient understanding of the calculation in front of them to be able to decide which method to use – mental, written or calculator, or a combination of these. When they do use a calculator, they should be able to draw on well-established skills of rounding numbers and calculating mentally to gain a sense of the approximate size of the answer, and have strategies to check and repeat the calculation if they are not sure whether it is right. These skills do not happen by accident. They need to be taught in mathematics and applied across the curriculum whenever calculators are used.

Proportional reasoning

Throughout Key Stage 3 pupils will extend their understanding of the number system to positive and negative numbers and, in particular, to fractions and their representations as terminating or recurring decimals.

Fractions, decimals, percentages, ratio and proportion are different ways of expressing related ideas and relationships. The connections start to be established in Key Stage 2, particularly the equivalence between fractions, decimals and percentages. The ideas of ratio and proportion, and the relationship between them, should be a strong feature of work in Key Stage 3. By the end of the key stage, pupils should be able to solve problems involving fractions, decimals, percentages, ratio and proportion, and their interconnections.

After calculation, the application of proportional reasoning is the most important aspect of elementary number. Proportionality underlies key aspects of number, algebra, shape, space and measures, and handling data. It is also central in applications of mathematics in subjects such as science, technology, geography and art. The study of proportion begins in Key Stage 2 but it is in Key Stage 3 where secure foundations need to be established.

Problems involving proportion are often solved by informal methods, particularly when the numbers involved are easy to deal with mentally. But it is important to teach methods that can be applied generally. For example, the unitary method is useful for solving problems involving proportion, and multiplicative methods involving fractions or decimals are useful for solving percentage problems.

When you are teaching proportional reasoning:

- emphasise the language and notation of ratio and proportion, and the links to fractions, decimals and percentages;
- teach pupils specific methods for solving proportion problems so that they do not remain dependent on informal approaches;
- help pupils to understand *what* they are calculating: for example, a distance divided by a time gives a speed – an example of a rate; but a distance divided by another distance gives a scale factor or multiplier – a dimensionless number;
- make explicit links between ideas of proportionality in number, algebra, shape, space and measures, and handling data.

In algebra, direct proportion is viewed as a linear relationship of the form $y = mx$. The graphical representation of this equation helps pupils to visualise ideas such as rate of change and gradient. The algebraic representation of a proportion (e.g. $a : b = c : d$ or $a / b = c / d$) underpins a general method for solving problems.

In shape, space and measures, proportionality arises when enlargement by different scale factors is considered. Scaling has a wide range of applications, for example, in maps, plans and scale drawings. Similar figures have sides or dimensions that are in proportion. Recognition of the similarity of all circles leads to an understanding that the circumference is directly proportional to the diameter, while awareness of the similarity of triangles with the same angles leads to an understanding of trigonometry.

In statistics, proportions are often calculated when data are interpreted and inferences drawn. Proportions are also used when probabilities are estimated or calculated based on outcomes that, in theory, are equally likely.

Features of number in Key Stage 3

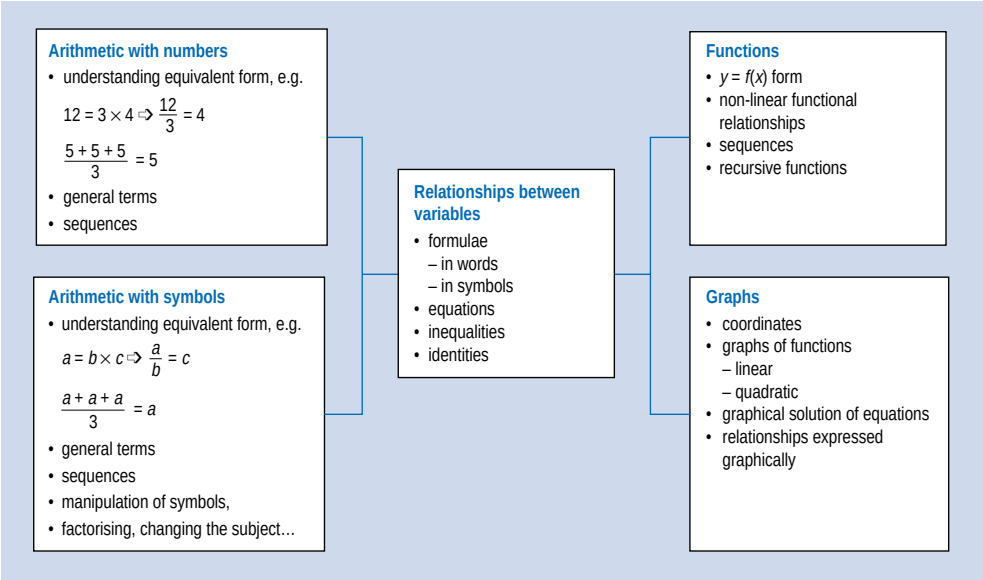
To summarise, the distinctive features of number in Key Stage 3 are:

- developing understanding of the number line;
- building on the *approach to calculation* developed in Key Stages 1 and 2, which emphasises mental methods and gradually refined written methods, extending to calculations with fractions, decimals and percentages;
- developing effective use of *calculators*, including choosing appropriate methods for estimating, calculating and checking;
- developing *proportional reasoning*, including making links to algebra, shape, space and measures, and handling data.

Introducing and developing algebra

Algebra in Key Stage 3 is generalised arithmetic. Its origins lie in arithmetic, in the art of manipulating sums, products and powers of numbers. The same rules are seen to hold true for all numbers, of whatever type, so it becomes possible to generalise the rules with letters in place of numbers. Indeed all numerical entities, coefficients as well as unknowns, can be represented by letters. This insight releases in due course the full power of algebra.

The diagram below is drawn from the Royal Society and Joint Mathematical Council's publication *Teaching and learning algebra pre-19*. The lines showing the links between topics are the most significant aspect.



Algebra in Years 7 to 9 includes equations, formulae and identities, and sequences, functions and graphs. You need to stress the links between these topics and with arithmetic. Letters do not represent quantities like length or cost; they represent numbers. Pupils will have spent much time manipulating numbers in Key Stages 1 and 2, and you can build on their experience. For example, in response to the question: '$6 \times 8 = 48$. What can you deduce from this?', a Year 6 class might suggest:

$8 \times 6 = 48$	$8 = 48/6$	$80 \times 6 = 480$	$16 \times 6 = 96$
$8 \times 6 + 1 = 49$	$6 = 48/8$	$80 \times 60 = 4800$	and so on

In Key Stage 3, this can be extended into the simple manipulation of equations, referring back regularly to the number examples. For example, $ab = c$ implies:

$ba = c$ $\quad b = c/a$ $\quad a = c/b$ $\quad 2ab = 2c$ $\quad 2ab + 1 = 2c + 1$ $\quad \ldots$

You can draw other examples from pupils' experience of multiplication based on the distributive law. By encouraging pupils to record their mental calculations in algebraic form by making use of brackets, you can help them to generalise. For example, you can draw parallels between:

$6 \times 42 = 6(40 + 2) = 6 \times 40 + 6 \times 2 = 240 + 12 = 252$

and: $a(b + c) = ab + ac$

or between: $147 - 99 = 147 - (100 - 1) = 147 - 100 + 1 = 48$

and: $a - (b - c) = a - b + c$

Equations, formulae and identities

The initial approach to manipulating number statements needs to extend into a set of rules for solving equations, under a general heading: 'do the same to both sides of the equation'. Pupils' developing skills in solving equations will be dependent on their ability to add, subtract, multiply and divide directed numbers and to simplify expressions and collect like terms. Although to begin with you will frequently refer to number examples, in time pupils will appreciate that expressions and equations can be manipulated in their own right according to given rules and conventions.

The generalised algebraic use of brackets, multiplying out of a single term over a bracket, and the inverse process of taking out a common factor, can also develop from examples with numbers. Multiplying out a pair of brackets can be based on the 'grid method' of multiplication and developed into an algebraic process. The most able pupils can attempt the reverse process of factorisation but for most pupils this will be an activity for Key Stage 4.

By the end of the key stage, pupils need to understand that a statement such as:

$$(x + 1)(x - 1) = x^2 - 1$$

is an identity that is true for every value of x, unlike an equation, where the purpose is to find the value(s) of x that alone satisfy the equation.

Sequences, functions and graphs

Many applications of algebra involve finding a formula that generates the general term of a sequence: for example, in predicting the number of matchsticks needed for a certain pattern, or the number of paving slabs for the border of a rectangular pond. It is important for pupils to justify their formulae from physical patterns, rather than merely from number sequences, since this allows them to 'prove' their solutions, not just illustrate or verify them. The apparently different but equivalent formulae that arise from alternative ways of looking at the problem can help pupils to understand equivalent algebraic expressions.

Functions and graphs can be taught and learned in tandem. At Key Stage 3 the main emphasis is on linear functions and their graphs. A graphical calculator, or graph plotting software, has an important role since it helps pupils to learn from exploring problems.

Features of algebra in Key Stage 3

To summarise, the distinctive features of algebra in Key Stage 3 are:

- developing understanding that algebra is a way of *generalising* from arithmetic, from particular cases or from patterns and sequences;
- providing regular opportunities to *construct* algebraic expressions and formulae and to *transform* one expression into another – collecting like terms, taking out common factors, working with inverses, solving linear equations;
- using opportunities to *represent* a problem and its solution *in tabular, graphical or symbolic form*, using a graphical calculator or a spreadsheet where appropriate, and to relate solutions to the context of the problem;
- developing *algebraic reasoning*, including an appreciation that while a number pattern may suggest a general result, a proof is derived from the structure of the situation being considered.

Shape, space and measures

Geometry in Key Stage 3 is the study of points, lines and planes and the shapes that they can make, together with a study of plane transformations. A key aspect is the use and development of deductive reasoning in geometric contexts. Geometrical activities can be linked to accurate drawing, construction and loci, and work on measures and mensuration. By ensuring that pupils have a range of suitable experiences you can develop their knowledge and understanding of shape and space and their appreciation of the ways that properties of shapes enrich our culture and environment.

Geometrical reasoning

Pupils can be aware of and use geometrical facts or properties that they have discovered intuitively from practical work before they can prove them analytically. The aim in Key Stage 3 is for pupils to use and develop their knowledge of shapes and space to support geometrical reasoning. For example, they need to appreciate that tearing the corners off a triangle and placing them side by side at best indicates that the angle sum of a triangle is approximately 180°, and that however many particular cases they can find of triangles with an angle sum of 180°, this does not prove the general case.

In Key Stage 3, you can build on pupils' experience and the practical demonstrations and explanations that have sufficed in Key Stages 1 and 2. Teach them to understand and use short chains of deductive reasoning and results about alternate and corresponding angles to reach a proof. Later, pupils should be able to explain why the angle sum of any quadrilateral is 360°, and to deduce formulae for the area of a parallelogram and of a triangle from the formula for the area of a rectangle. These chains of reasoning are essential steps towards the proofs that are introduced in Key Stage 4.

Appreciation of shape and space

Geometry cannot be learned successfully solely as a series of logical results. Pupils also need opportunities to use instruments accurately, draw shapes and appreciate how they can link together, for example, in tessellations. In Key Stage 3, it is vital to distinguish between the imprecision of constructions which involve protractors and rulers, and the 'exactness in principle' of standard constructions which use only compasses and a straight edge. Geometrical reasoning can show pupils why construction methods work, for example, the method to construct a perpendicular bisector of a line segment.

Practical work with transformations will produce interesting problems to solve as well as helping pupils to understand the topic more fully. Urge them to visualise solutions to problems such as: 'When a triangle is rotated through 180° about the mid-point of one side, what shape do the original and final triangles form?' Linking geometry to subjects such as art, through symmetry or tessellations, or religious education, perhaps through a study of the properties of Islamic patterns or cathedral rose windows, offers good opportunities to develop creativity. By encouraging pupils to speculate why the properties they have found hold true you can strengthen their reasoning skills.

Use of ICT

ICT offers good opportunities to develop geometrical reasoning and an appreciation of shape and space. For example, pupils can use the programming language Logo to explore properties of plane shapes, such as the exterior angles of polygons. With dynamic geometry software, they can use rapid geometric drawing to explore a condition such as 'one pair of opposite angles of a quadrilateral is equal', and discover the special

circumstances under which the condition is true. More able pupils may be able to prove their conjectures analytically, but the formal use of congruent triangles is often needed, and for most pupils this will be tackled in Key Stage 4.

Measures and mensuration

Pupils in Key Stage 3 need to develop their awareness of the relative sizes of units, converting between them, and using the rough equivalence of common imperial and metric units. Towards the end of the key stage, they need to become familiar with compound measures such as speed or density. Help them to appreciate the imprecision of measurement and to recognise the accuracy to which measurements can be stated. Draw as far as possible on their practical experience of measures in other subjects, particularly design and technology, science, geography and physical education.

Work on perimeter, area and volume will extend to a range of shapes, including rectangles, parallelograms, circles, cuboids and prisms. A project such as 'design a swimming pool' allows pupils to exercise imagination, practise calculations of length, area and volume, and experience working with larger numbers and units. The heart of work on mensuration will be with triangles which, at the end of the key stage, will extend to Pythagoras' theorem and similarity, leading on to trigonometry. As far as possible, the relevant formulae for calculating perimeters, areas and volumes should be explored and justified logically, not simply stated as facts. The use of formulae can be linked to work in algebra, and can be enhanced by the use of spreadsheets and graphical calculators.

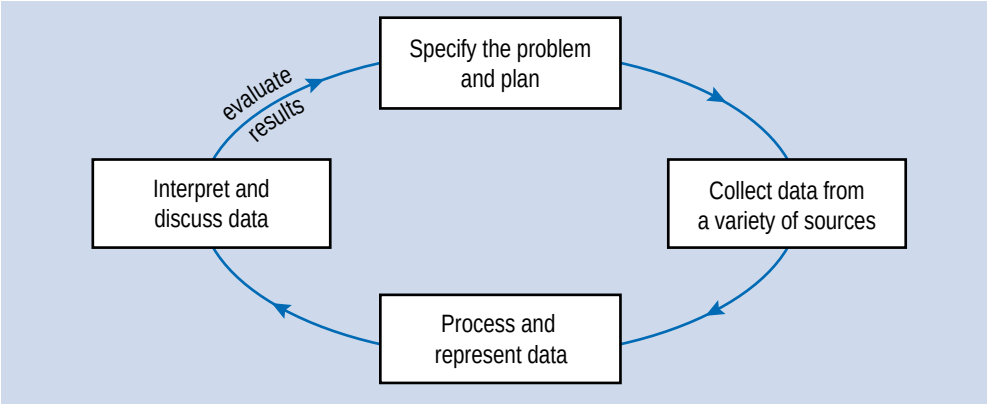
Features of shape, space and measures in Key Stage 3

To summarise, the distinctive features of shape, space and measures in Key Stage 3 are:

- developing *geometrical reasoning* and *construction skills*, and an appreciation of logical deduction;
- developing *visualisation* and *sketching* skills, including a dynamic approach to geometry, making use of ICT and other visual aids;
- developing awareness of the degree of accuracy of measurements.

Handling data

Handling data is described by the cycle shown in the diagram.



Data handling is best taught in a coherent way in the context of real statistical enquiries so that teaching objectives arise naturally from the whole cycle. As an enquiry develops, you will need to reinforce and develop certain skills by direct teaching of particular objectives.

It is easier to make sure that problems are relevant if at least some of the enquiries are linked to other subjects. For example, a question can be formulated and data collected in geography, with mathematics lessons concentrating on processing, representing and interpreting the data. The other subject can make further interpretations and consider the implications of what has been discovered. The supplement of examples in this Framework suggests a variety of possible projects, many of them linked directly to QCA's schemes of work for other subjects, including science, geography and physical education, with further possibilities in history, religious education, languages and personal, social and health education. Your choice of project will obviously depend on the collaboration that is possible between departments in your own school.

As pupils move through Key Stage 3, the cross-curricular aspects of data handling become more important. It is usually best for a cross-curricular enquiry to be defined in the other subject, but good preparation is needed to check that the mathematical skills, techniques and representations that pupils need to learn next are likely to arise. In Year 7, much of the work may take place in mathematics lessons, with small sets of data that pupils can generate readily from simple experiments and easily accessible secondary sources. In Year 9, pupils should engage with large sets of real data from a much wider range of sources and contexts. After all, their GCSE coursework in Key Stage 4 may require them to undertake a major statistical investigation, with supporting ICT. The experience of working with real data in Key Stage 3 is an important preparation.

Primary and secondary sources

Give pupils experience of collecting and using primary data from, for example, questionnaires or results of an experiment, and secondary data from published sources, including reference materials, ICT databases and the Internet. Plan carefully how to balance and use the various sources across the key stage.

Real data present problems that 'textbook' or contrived data can skirt around, such as the accuracy of recording or how to deal with data that are ambiguous. The sizes of numbers can be problematic, either because they are large or, in the case of a pie chart, because they are not factors of 360. The time to process and represent real data is likely

to be greater than with textbook examples but by using them pupils will have gained useful skills that can be transferred to other investigations.

Probability

Pupils will have met some of the language of probability in Key Stage 2 but will have had little experience of quantitative probability. Pupils need to understand that probability is a measure of what might happen. Help them to reason out what the probability is for simple cases by considering all the possible outcomes for particular events.

The data-handling cycle illustrated on page 18 applies also to probability. There are two aspects to develop in tandem through this cycle: probability described by the proportion of successes in an experiment, and probability derived from theoretical considerations. As they compare their practical and theoretical results, pupils will begin to sense that as the number of trials becomes very large the proportion of successes converges to the theoretical probability.

Features of handling data in Key Stage 3

To summarise, the distinctive features of handling data in Key Stage 3 are:

- basing work on *purposeful enquiry*, using situations of interest and relevance to pupils and making appropriate links to other subjects;
- placing an emphasis on *making inferences* from data, drawing on a range of secondary sources to ensure that samples are sufficiently large;
- using *ICT* as a powerful source of data, and as a means of processing data and simulating situations.

Using and applying mathematics and thinking skills

Thinking skills underpin using and applying mathematics and the broad strands of problem solving, communication and reasoning. Well-chosen mathematical activities will develop pupils' thinking skills. For example, you might devote occasional stand-alone lessons to an investigation of a problem. Used well, this approach can focus pupils' attention on the 'using and applying' or thinking skills that they have used so that they can apply these skills more generally in their mathematics work. But the main approach to using and applying mathematics and thinking skills is to integrate them within everyday teaching, thereby helping pupils to make the connections in learning associated with success in mathematics. For example, the unit of work on algebra described on page 52 includes some 'using and applying' objectives.

You can introduce problem solving, applications of mathematics and the use of reasoning and thinking skills at many points in a unit of work. A problem can serve as an introduction, to assess pupils' prior knowledge or to set a context for the work; it can be used to provide motivation for acquiring a skill; or it can be set as a class activity or as homework towards the end of a topic, so that pupils use and apply the mathematics they have been taught.

Pupils need to be able to select the mathematics required to solve a problem and to recognise that an idea that they meet in one strand of mathematics can be applied in another. A good 'diet' will include:

- problems and applications that extend content beyond what has just been taught;
- familiar and unfamiliar problems in a range of numerical, algebraic and graphical contexts, some with a unique solution and some with several possible solutions;
- activities that develop short chains of deductive reasoning and concepts of proof in algebra and geometry;
- occasional opportunities to sustain thinking by tackling more complex problems.

There are five categories of thinking skills embedded in the National Curriculum: information processing skills, enquiry skills, creative thinking skills, reasoning skills and evaluation skills. The contribution of mathematics to these skills is drawn directly from using and applying mathematics.

Information processing skills

These enable pupils to find and organise relevant information, to compare and contrast it, and to identify and analyse relationships. For example, pupils acquire information processing skills when they process the information in a 2-D picture of a 3-D object, or when they investigate what factors influence the distribution of grass and non-grass plants on the school field. They compare and contrast when, for example, they classify quadrilaterals, order a set of decimals, or consider two different representations of the same set of data.

The National Curriculum for using and applying mathematics states that pupils should:

- collect data from a variety of suitable sources, including experiments and surveys, and primary and secondary sources, including the Internet;
- decide what statistical analyses are needed;
- process and represent the data, using ICT as appropriate, turning raw data into usable information that gives insight into the problem or enquiry;
- interpret data and draw conclusions from them.

Enquiry skills

Enquiry lies at the heart of mathematics. Enquiry skills enable pupils to ask questions, define questions for enquiry, plan research, predict outcomes, anticipate consequences, and draw conclusions. Central to enquiry is an ability to see connections between different aspects of mathematics and thus open up further ways of tackling a problem (see 'Making connections', page 46).

Pupils develop enquiry skills when they modify a problem they have solved to create a new problem: for example, when they find the smallest number with exactly three factors, then extend this to four factors, or other numbers of factors. They use enquiry skills when they find and eliminate alternatives: for example, when they explore how many different shapes can be made with five identical squares, and decide what to do with reflections or rotations. They practise enquiry skills in cross-curricular statistical surveys: for example, by examining possible indicators for economic development and deciding which of them are likely to highlight differences between countries most clearly.

The National Curriculum for using and applying mathematics requires pupils to:

- pose problems and plan how to solve them;
- predict outcomes;
- decide what data or information to collect, the degree of precision or accuracy required, and the inferences they will be able to draw;
- choose the appropriate mathematics and resources to use, including ICT;
- interpret and discuss representations of data and results of analyses, looking for patterns and relationships, and explaining and justifying how they reached conclusions;
- identify what further information is needed to pursue a supplementary enquiry.

Creative thinking skills

These enable pupils to generate and develop ideas, to hypothesise, to apply imagination, and to seek innovative alternatives. For example, pupils need to think creatively when they visualise the path of a moving point or how a shape might change as slices are taken from it, when they look for pairs of numbers whose sum equals their product, or when they formulate algebraic expressions to make this addition square work:

	?	**?**
?	$3a + 2b$	$a + 4b$
?	$4a + 3b$	$2a + 5b$

The National Curriculum for using and applying mathematics requires pupils to:

- select and combine known facts and problem-solving strategies in creative ways to solve problems, using alternative approaches to overcome difficulties;
- represent problems and solutions in numerical, algebraic, geometric or graphical form, moving from one to another to gain a different perspective on the problem and to identify similarities and differences;
- explore connections in their mathematical work;
- explore, identify, and use pattern and symmetry in mathematical contexts;
- visualise and use mathematical imagery;
- conjecture, hypothesise, and ask questions such as 'What if …?' or 'Why?';
- investigate whether particular cases can be generalised further.

Reasoning skills

These enable pupils to give reasons for opinions and actions, to infer and deduce, to make informed judgements and decisions, and to use precise language to reason. Pupils apply reasoning skills when, for example, they investigate how to draw two straight lines from the vertex of a square to divide the square into three equal parts and then prove analytically that their construction holds, or when they look for examples of pairs of unit fractions whose sum is another unit fraction. They apply reasoning when they argue, for example, that it is necessary but not sufficient that a multiple of 4 is an even number, or that a square is a trapezium but that a trapezium is not necessarily a square.

The National Curriculum for using and applying mathematics states that pupils should:

- use step-by-step deduction and efficient techniques for solving a problem, including breaking down complex problems into simpler steps or a series of tasks, and working systematically;
- present concise, reasoned arguments, explaining and justifying inferences, deductions and conclusions, using mathematical notation, symbols and diagrams correctly and consistently;
- identify exceptional cases or counter-examples that do not accord with an argument and explain why;
- distinguish between practical demonstration and proof, and between conventions, definitions and derived properties.

Evaluation skills

These enable pupils to develop and apply evaluation criteria and to judge the value of information and ideas. For example, pupils use evaluation skills when they compare mental methods of calculation to judge which is the most efficient, or when they evaluate whether the use of a calculator is appropriate. They use evaluation skills when they generate a number sequence in a practical context and then consider the best way to describe and express the general term. They might evaluate the questions used in a statistical survey to compare attitudes to fairly traded goods and decide that the questions used to explore attitudes are crude compared with the complexity of personal beliefs.

The National Curriculum for using and applying mathematics requires pupils to:

- review progress as they work;
- move from one form of representation to another to gain different perspectives on a mathematical problem;
- evaluate the effectiveness of their chosen methods, techniques and problem-solving strategies, and the resources they have chosen to use;
- check and evaluate their solutions, including the accuracy of their calculations, analyses and results;
- consider anomalies in data or measurements and try to explain them;
- examine critically, improve, then justify their choices of mathematical presentation.

There are more illustrations of the ways that thinking skills are embedded in using and applying mathematics on pages 26–35 of the supplement of examples.

Mathematics across the curriculum

Mathematics contributes to and draws from many subjects and aspects of the curriculum. You can help pupils to appreciate the importance of mathematics in their lives by making these links explicit. For example, if you know how mathematics is applied in other subjects, and ask colleagues to provide examples of applications to use in mathematics lessons, you will be able to provide examples and contexts which pupils know and understand.

It will help pupils to use mathematics in other subjects if they are first taught the mathematics in mathematics lessons. To do this, you need to know what mathematics pupils will be expected to know and use in other subjects, particularly science, design and technology, and geography.

Good liaison should help to ensure that all staff use common approaches that mirror those used in mathematics lessons. For example, the approach to calculation should be the same in science as in mathematics. You need to explain:

- the use of mental and informal written methods, especially with lower attaining pupils;
- the expectation that pupils should add and subtract pairs of two-digit numbers mentally;
- how and when calculators should be used.

Besides the approach to calculation, you need to agree across departments:

- the use of units;
- the mathematical notation and terms to be used;
- algebraic and other mathematical techniques, such as how algebraic expressions are to be simplified or how equations are to be solved;
- how graphs are to be represented;
- how and when ICT resources such as graph plotters or graphical calculators will be used to support mathematics.

Mathematical skills can be consolidated and enhanced when pupils have opportunities to apply them across the curriculum. Some opportunities to link mathematics to work in other subjects are as follows.

English and literacy The National Curriculum statement on language suggests three areas to include in all subject teaching:

- general accuracy in using language – spoken, written and read;
- technical terms and concepts appropriate to the subject;
- awareness of patterns of language.

In mathematics, general accuracy in using language can be promoted through: interpreting questions posed orally or in writing; clarifying the precise meaning of words or mathematical terms; discussing the essential ideas identified in the questions and interpreting them to identify the mathematical content. Awareness of patterns of language can be developed by asking pupils to explain, argue and present their conclusions to others, and by drawing their attention to the statements involved in mathematical reasoning and proof, such as *if … then*, *because*, *therefore*, *implies…* The technical terms and concepts used in mathematics will include the ideas of an *inverse*, of *equivalence*, *equality*, *proportionality*, *congruence*, *similarity*, *linearity*, and so on.

Science Almost every scientific investigation or experiment is likely to require one or more of the mathematical skills of classifying, counting, measuring, calculating, estimating, and recording in tables and graphs. Pupils will, for example, order numbers, including decimals, calculate means and percentages, use negative numbers when taking temperatures, decide whether it is more appropriate to use a line graph or bar chart, and plot, interpret and predict from graphs. They will explore rates of change in cooling curves and distance–time graphs, apply formulae and solve equations, for example, in problems on moments.

Art, design and technology Measurements are often needed in art and in design and technology. Many patterns and constructions in our own and other cultures are based on spatial ideas and properties of shapes, including symmetry. Designs may need enlarging or reducing, introducing ideas of multiplication, scale and ratio. The preparation of food involves measurement, working out times and calculating cost, frequently extending into calculations involving ratio and proportion.

Information and communication technology In ICT lessons, pupils will collect and classify data, enter them into data-handling software, produce graphs and tables, and interpret and explain their results. Their work in control will include the measurement of distance and angle. Spreadsheet skills, used in modelling and simulations, rely on the numeric, algebraic and graphical skills involved in constructing formulae and generating sequences, functions and graphs.

History and geography Discussing evidence in history or geography may involve measurement, estimation and approximation skills, and making inferences. Pupils will make statistical enquiries, for example, in analysing population data to explore and compare lifestyles; they will also use a wide range of measurements and rates of change. The study of maps includes the use of coordinates and ideas of angle, direction, position, scale and ratio.

Physical education and music Athletic activities use measurement of height, distance and time, and data-logging devices to quantify, explore, and improve performance. Ideas of counting, time, symmetry, movement, position and direction are used extensively in music, dance, gymnastics, athletics and competitive games.

Religious education, PSHE and citizenship Belief and likelihood in religious education, or risk assessment in PSHE, relate well to work in mathematics. The discussion of moral and social issues is likely to lead to the use of primary and secondary data and the interpretation of graphs, charts and tables, helping pupils to make reasoned and informed decisions and to recognise biased data and misleading representations. By applying mathematics to problems set in financial and other real-life contexts pupils will develop their financial capability and awareness of the applications of mathematics in the workplace.

Information and communication technology (ICT)

ICT includes calculators and extends to the whole range of audiovisual aids, including educational broadcasts and video film. The National Curriculum suggests points at which ICT can be used in mathematics. For example:

> *Generate functions from plots of data, for example, from a science experiment, using simple curve fitting techniques on graphic calculators, or with graphics software to explore the transformation of graphs.* The National Curriculum 1999

The main uses of ICT in mathematics in Key Stage 3 stem from:

- the use of calculators for calculating purposes (see page 12);
- small programs, such as number games or investigations in a particular context;
- programming languages, such as Logo or Basic, and the programming capabilities of graphical calculators;
- general-purpose software, particularly spreadsheets, but also databases;
- content-free mathematics software, such as graph plotters, dynamic geometry software and data-handling packages;
- ILS (Independent Learning Systems), which provide and manage practice in mathematical techniques tailored to the needs of individual pupils;
- graphical calculators and data-loggers;
- CD-ROMs and the Internet.

The supplement of examples includes some references to ICT. Small programs are not mentioned by name, since the focus in the supplement is on mathematical outcomes, not the resources that can be used to achieve them. Where appropriate, you could add references to the small programs and other ICT resources that you have access to.

ICT training and support The New Opportunities Fund, which will continue to 2003, will provide most teachers with about 30 hours of subject training on the use of ICT, tailored to their individual needs. To help identify training needs, the Teacher Training Agency (TTA) has produced a CD-ROM of four case studies of different mathematics lessons in Years 7 to 10, in which dynamic geometry software, a number game, a spreadsheet and graphical calculators are used. The case studies also show different ways of projecting screen images to whole classes.

The National Grid for Learning (NGfL) aims to connect all schools to the Internet. A major element of the NGfL is the Virtual Teachers' Centre, which includes useful information, resources and advice for mathematics teachers. There is more useful information, including reviews of software, on the websites of The Mathematical Association at www.m-a.org.uk and The Association of Teachers of Mathematics at www.atm.org.uk.

Funding for hardware and software The Standards Fund can be used to buy hardware and software. It also supports teachers' use of ICT for their professional development and for administrative work associated with teaching. You can buy:

- ICT equipment, including class sets of portable computing devices such as graphical calculators, whole-class teaching aids such as projection equipment, or equipment to help create teaching materials;
- technical training which complements but does not replicate the pedagogical training supported by the New Opportunities Fund (see above);
- software related to mathematics, including online resources and site licences.

Teaching strategies

The recommended approach to teaching is based on ensuring:

- sufficient regular teaching time for mathematics, including extra support for pupils who need it to keep in step with the majority of their year group;
- a high proportion of direct, interactive teaching;
- engagement by all pupils in tasks and activities which, even when differentiated, relate to a common theme;
- regular opportunities to develop oral, mental and visualisation skills.

Teaching time

In Key Stage 2, a daily one-hour mathematics lesson is provided, accounting for some 20% of the total teaching time. In Key Stage 3, the typical proportion of teaching time given to mathematics is 12%, equivalent to about three hours a week. School timetablers should aim to ensure that mathematics lessons in each year group are frequent and spread across the week, not bunched together.

Organisational models such as four lessons of 50 minutes are useful. They satisfy the principle of frequency, ensuring that pupils maintain and sharpen their mathematical skills through near daily contact. Other models, such as three 70-minute mathematics lessons, or six 35-minute lessons made up of double and single periods, need to be organised carefully, particularly where fortnightly timetables operate. Seventy minutes can be too long for pupils to maintain their concentration if the teaching is to be intensive and direct; on the other hand, a single 35-minute period offers too little time for ideas to be developed and consolidated in the main part of the lesson.

The focus on direct teaching

Aim to spend a high proportion of each lesson in direct teaching, often of the whole class, but also of groups and of individuals.

High-quality direct teaching is oral, interactive and lively, and will not be achieved by lecturing the class, or by always expecting pupils to teach themselves indirectly from books. It is a two-way process in which pupils are expected to play an active part by answering questions, contributing points to discussions, and explaining and demonstrating their methods and solutions to others in the class.

Good direct teaching is achieved by balancing different teaching strategies:

- **Directing and telling:** sharing your teaching objectives with the class, ensuring that pupils know what to do, and drawing attention to points over which they should take particular care, such as how to ensure that one step follows from another in a mathematical argument, the degree of accuracy to adopt when making a measurement, how to set out work or to label a graph…
- **Demonstrating and modelling:** giving clear, well-structured demonstrations: for example, showing how to solve an algebraic equation, interpret a graph or develop a rigorous mathematical argument; modelling mathematics using appropriate resources and visual displays: for example, using blackboard instruments to demonstrate a geometric construction, using a thermometer to model the use of

negative numbers, using a graphical calculator to find the solution to equations or dynamic geometry software to reflect a shape in a mirror line…

- **Explaining and illustrating:** giving accurate, well-paced explanations, and referring to previous work or methods: for example, explaining a method of calculation and discussing why it works, giving the meaning of a mathematical term, symbol or form of notation, explaining the steps in the solution to a problem, giving examples that satisfy a general statement, illustrating how the equation $y = 2x$ can represent different situations…
- **Questioning and discussing:** questioning in ways which match the direction and pace of the lesson to ensure that all pupils take part (if needed, supported by apparatus, a calculator or a communication aid, or by an adult who translates, signs or uses symbols); using open and closed questions, skilfully framed, adjusted and targeted to make sure that pupils of all abilities are involved and contribute to discussions; asking for explanations; giving time for pupils to think before inviting an answer; listening carefully to pupils' responses and responding constructively in order to take forward their learning; challenging their assumptions and making them think…
- **Exploring and investigating:** asking pupils to pose problems or suggest a line of enquiry, to investigate whether particular cases can be generalised, to seek counter-examples or identify exceptional cases; encouraging them to consider alternative ways of representing problems and solutions, in algebraic, graphical or diagrammatic form, and to move from one form to another to gain a different perspective on the problem…
- **Consolidating and embedding:** providing varied opportunities to practise and develop newly learned skills, through a variety of activities in class and well-focused homework; asking pupils either with a partner or as a group to reflect on and talk through a process; inviting them to expand their ideas and reasoning, or to compare and then refine their methods and ways of recording their work; encouraging them to use and apply their mathematical skills to solve mathematical problems across the curriculum…
- **Reflecting and evaluating:** identifying pupils' errors, using them as positive teaching points by talking about them and any misconceptions that led to them; discussing pupils' justifications of the methods or resources they have chosen; evaluating pupils' presentations of their work to the class; giving them oral feedback on their written work…
- **Summarising and reminding:** reviewing during and towards the end of a lesson the mathematics that has been taught and what pupils have learned; identifying and correcting misunderstandings; inviting pupils to present their work and picking out key points and ideas; making links to other work in mathematics and other subjects; giving pupils an insight into the next stage of their learning…

Direct teaching and good interaction are as important in group work and paired work as they are in whole-class work. Organising pupils as a 'whole class' for a significant proportion of the time helps to maximise their contact with you so that every pupil benefits from the teaching and interaction for sustained periods.

A typical lesson

Secondary schools work to different time constraints so the structure and timing of mathematics lessons will differ. The outline structure of a three-part lesson described below is highly recommended since it can be adapted to different circumstances. It provides 'a beginning, a middle and an end' in which you explain to pupils and prepare them for what they are to learn, teach it to them, then help them to recognise what they have achieved. It allows a variety of patterns of teaching methodology and organisation, depending on a lesson's objectives and its position in a series of lessons.

A typical mathematics lesson

- **An oral and mental starter** (about 5 to 10 minutes)
  whole-class work to rehearse, sharpen and develop mental skills, including recall skills, and visualisation, thinking and communication skills
- **The main teaching activity** (about 25 to 40 minutes)
  combinations of teaching input and pupil activities
  work as a whole class, in pairs or groups, or as individuals
  interventions to identify and sort out misconceptions, clarify points and give immediate feedback
- **A final plenary** to round off the lesson (from 5 to 15 minutes)
  whole-class work to summarise key facts and ideas and what to remember, to identify progress, make links to other work, discuss the next steps, set homework

This outline structure is not a mechanistic recipe to be followed. Use your professional judgement to determine the activities, timing and organisation of the beginning, middle and end of the lesson to suit its objectives. For example, with four 50-minute lessons each week, you might have a starter of 15 minutes on one day, with a brisk two or three minutes on the remaining days. In the longer starter, pupils might enter the room and start to work on a challenging problem posed on the board, with mental and oral work, based on their initial thoughts on the problem, taking place after five or ten minutes.

In the main part of the lesson, in particular, there is scope for considerable variety and creativity, with a different interplay of work with the whole class, groups, pairs and individuals on different days, although each lesson should include direct teaching and interaction with the pupils, and activities or exercises that pupils do.

Overall, aim for a high proportion of work with the whole class, but there may be more in some lessons than in others. For example, at the start of a new unit of work you might need more time for demonstration, explanation and discussion with the whole class, interspersed with very short exercises for pupils; the plenary may be very short. On the other hand, when you have identified general errors or misunderstandings during the main part of a lesson, you may need several mini-plenaries during the lesson to sort them out, as well as a final summing-up. Later in a unit of work, pupils might start the main part of a lesson by continuing to work in pairs on a previous problem; when they have had time to re-enter the problem, you might hold a mini-plenary with the whole class to share ideas, highlight important results, and structure work from there on in the lesson.

At the end of a unit of work it can be useful to use the plenary to look back with the whole class over a number of lessons to draw together what has been learned and to identify the key points and methods that you want pupils to remember and use in the future. For this kind of plenary, you may need a much longer time than usual.

The oral and mental starter

Experience in the primary Literacy and Numeracy Strategies has shown that a lively, interactive opening to a lesson with a sharp focus on specific objectives will engage pupils' attention and have a more lasting effect than periodic 'skills' or 'numeracy' lessons. Starter activities are popular with the whole ability range but are especially beneficial for classes where pupils have not yet secured a good range of oral, mental and visualisation skills. For example, you could use this time to:

- rehearse previously taught skills in a variety of lively ways;
- focus on the skills needed in the main part of the lesson.

Starter activities involve much more than testing pupils' mental recall or reading out homework answers! You need to plan carefully a balanced programme of starter activities so that it provides coherence and progression over time. Besides factual recall, a programme of short oral and mental activities might include:

- developing and explaining mental calculation strategies, including figuring out new facts from known facts;
- applying calculation skills in algebra, for example, to substitute numbers in expressions or to solve simple equations;
- developing estimation skills;
- interpreting data;
- visualising and describing shapes, movements and constructions;
- developing mathematical vocabulary;
- developing the ability to generalise, reason and prove.

A starter of this kind will be more effective if you:

- get off to a clear start and maintain a brisk pace;
- devise strategies to absorb 'stragglers' who arrive from different classrooms;
- ensure that all pupils can see you easily and can and do take part;
- prepare a good range of open and closed questions to put to the whole class, with some to ask particular individuals, pairs or groups;
- use pupils' responses to make an informal assessment of their progress;
- brief any support staff to give discreet help to any pupils who need support.

The main teaching input and pupil activities

The main part of the lesson provides time to:

- introduce a new topic, consolidate previous work or develop it;
- develop vocabulary, use correct notation and terms and learn new ones;
- use and apply concepts and skills;
- assess and review pupils' progress.

This part of the lesson is more effective if you:

- make clear to the class what they will learn;
- make links to previous lessons, or to work in other subjects;
- give pupils deadlines for completing activities, tasks or exercises;
- maintain pace, making sure that this part of the lesson does not over-run and that there is enough time for the plenary.

When you are teaching the **whole class** it helps if you:

- demonstrate and explain using a board, flip-chart, computer or OHP;
- highlight the meaning of any new vocabulary, notation or terms, and encourage pupils to repeat these and use them in their discussions and written work;
- involve pupils interactively through carefully planned and challenging questioning;

- ask pupils to offer their methods and solutions to the whole class for discussion;
- identify and correct any misunderstandings or forgotten ideas, using mistakes as positive teaching points;
- ensure that pupils with particular needs are supported effectively.

When pupils are working on tasks in **pairs**, **groups** or as **individuals** it helps if you:

- keep the whole class busy working actively on problems, exercises or activities related to the theme of the lesson;
- encourage discussion and cooperation between pupils;
- where you want to differentiate, manage this by providing work at no more than three or four levels of difficulty across the class;
- target a small number of pairs, groups or individuals for particular questioning and support, rather than monitoring them all;
- make sure that pupils working independently know where to find resources, what to do before asking for help and what to do if they finish early;
- brief any supporting adults about their role, making sure that they have plenty to do with the pupils they are assisting.

The plenary session

The plenary is an opportunity to round off and summarise the lesson, so that pupils focus on what was important, what they have learned and the progress they have made. It is a time when you can relate mathematics to their work in other subjects: for example, how their work on calculation will be used in science, or how their measuring skills will be practised in physical education or design and technology.

You can use this part of the lesson to:

- draw together what has been learned, summarise key facts, ideas and vocabulary, and stress what needs to be remembered;
- generalise some mathematics from examples generated earlier in the lesson;
- go through a written exercise pupils did individually during the lesson, so that you can question them about it, assess it informally and rectify any remaining misconceptions or errors;
- make links to other work and what the class will go on to do next;
- highlight the progress made and remind pupils about their personal targets;
- set homework to extend or consolidate class work and prepare for future lessons.

The plenary part of the lesson will be more effective if you:

- have a clear idea of its purpose and what you want to achieve in it.

Homework and out-of-class work

You should set regular homework, modifying its presentation for any pupils who need this. Homework will usually be short and focused, with varied and interesting tasks that motivate pupils, stimulate their learning and foster different study skills. The work will often be a short task, either an exercise or an open-ended small task, which helps to consolidate and develop work done in class. Occasionally, a longer homework might be set that requires pupils to follow a line of enquiry or to use the mathematics which has been taught over the last few weeks, perhaps combined with work in another subject.

By marking homework promptly you can glean useful diagnostic information on who has learned what and who needs extra support.

Not every piece of homework needs to be written work that has to be marked, though it still needs an acknowledgement to show pupils that their efforts are valued. For example:

- You could discuss a problem briefly in the plenary part of a lesson and ask pupils to tackle it in preparation for the next lesson. This could start with sharing and refining methods and solutions, which are then used to inform the main teaching activity, when you give similar or linked problems to each group.
- In preparation for the next lesson you could ask pupils to gather data from secondary sources or the Internet, or they could take a set of measurements.
- You could use the plenary to introduce a puzzle or activity that helps pupils to practise their recall of facts. At the start of the next lesson you could focus on the recall of these facts so that you can see through your interactions with the pupils which of them have good recall and which need some extra support.

Inclusion and differentiation

Try to ensure that, as far as possible, pupils work together through the planned programme for their class so that all of them are included in each unit of work, participate fully in lessons and benefit from the discussion and interaction with their teacher and their peers.

Many secondary schools organise pupils into ability sets for mathematics, since planning tends to be easier if the attainment gap in a class is not too wide. The success of setting depends on close teamwork, cooperative planning and careful monitoring by mathematics staff to make sure that pupils can move from set to set as their progress demands and that expectations for all pupils are suitably high; lower expectations are not justified simply because pupils are in a 'lower set'. Teachers of higher sets may well base their pupils' work on a programme for an older age group: for example, after the first few weeks of the autumn term, a Year 7 class in a selective school, or a top set in a comprehensive school, is likely to follow a programme based largely on the Year 8 objectives. Teachers of lower sets may need to work mainly from objectives in the teaching programmes for a younger age group, while keeping in mind the objectives for the appropriate year.

Even where pupils are organised in ability sets for mathematics, there can still be a range of attainment in a class. There are several ways of meeting the needs of mixed ability classes or of ability sets with a spread of attainment. The first step is to establish a classroom climate where all pupils feel that they can contribute, and which secures their motivation and concentration. The next step is to adopt teaching and organisational strategies to keep all pupils involved and suitably challenged, while giving them maximum opportunity to interact with their teacher. This includes providing appropriate support, aids or interventions to give particular pupils access to the planned programme and to keep any who might fall behind in step with the rest of their class.

Differentiation in whole-class oral and mental work

In oral work and mental starters with the whole class, you could begin with some questions that all pupils can manage in order to get them involved and interested. When you direct questions to the whole class, maintain pace but build in enough 'wait time' for pupils to think or discuss with a partner before answering. Where possible, use open questions that allow more pupils to respond. You can also target an individual or group with particular challenges suited to their abilities or needs, such as pupils at the earlier stages of learning English, very able pupils, or pupils with special educational needs. You may sometimes need to give particular pupils some discreet help with the vocabulary or the method.

Closed questions such as 'What is 30×125?' can be opened up by discussing the methods used. Some pupils may first do 3×125 and then multiply the answer by 10, others may do 30×120 first and then add 30×5. Jotting down interim steps of these calculations may help some while others may do the entire calculation mentally. The answer $30 \times 125 = 3750$ can lead to discussion of other statements that can now be deduced, such as $3750 \div 125$, 0.3×1.25 or $37.5 \div 1.25$.

Differentiation in written work and homework

Differentiated group work is another way of catering for a range of attainment, particularly in mixed ability classes. To plan differentiated tasks around the same mathematical topic at, say, three levels, you can draw on related teaching objectives for that topic from the programmes for the appropriate age group, and from the next and the previous age groups.

Written tasks and homework can be adapted to suit particular needs without varying either the task or the level of difficulty: for example, by presenting them in enlarged print or on audio tape. Some pupils may need tasks broken down into structured steps. For other pupils, you may need to identify and explain key words, or use extra diagrams or illustrations for particular points. There may also be pupils who continue to need apparatus to support their thinking and suitable resources need to be on hand for them to choose from. At the same time, you need to encourage the pupils to dispense with support materials when you and they judge that they are ready.

Some pupils work faster than others, perhaps because they use short cuts or are generally more confident and more able. They may need to do fewer examples and move on to extension or enrichment tasks linked to the theme of the lesson so that they use and apply their skills in more challenging contexts. Others may need longer to practise and consolidate what they have been learning and need examples at each level of difficulty. Some pupils take longer to record or present their work; for them, it is more beneficial to do selected key questions from each section of an exercise than to work through from the beginning.

Pupils who are very able

The yearly teaching programmes in this Framework are targets for the majority of pupils in the year group (see links to National Curriculum levels, page 4). Able pupils deal with abstract mathematics more readily than other pupils do. They will progress more quickly through these programmes and will need extension and enrichment activities to develop the breadth of their mathematics and the depth of their thinking. They can be stretched by being given extra challenges and harder problems to do when other pupils are consolidating, by offering occasional differentiated group work, and by drawing work from the teaching programmes for older pupils, and the higher programmes of study for Key Stage 4. Homework also provides opportunities to set suitably challenging tasks.

Sometimes special arrangements are made for very able pupils. For example, where there are sufficient numbers of them they may be taught in an express set. There they can benefit from discussion with other pupils working at a similar, more advanced level. Where this is not possible, very able or gifted pupils who are markedly ahead of the rest of their class can follow individualised programmes at appropriate times, with far fewer practice examples and many more challenging problems to tackle, including work that draws on other subjects. Of course, they still need some teaching to ensure that they understand what they have read and know how to present their work.

All pupils, but especially the very able, need to carry out sustained mathematical investigations both in school and, where appropriate, continued at home. There are many good publications to support this kind of work, including materials from problem-solving websites.

Pupils who need to catch up

A large and important group of pupils face relatively minor difficulties in learning, reflected in mathematical attainment just below the level expected at entry to Year 7. Some may have been disadvantaged by factors associated with circumstances at home. Some may have been moved to a number of different schools, or have gaps in learning resulting from missed or interrupted schooling: for example, travellers, refugees, those in care or those with medical conditions. Others may have misconceptions remaining from earlier work, or have weaker literacy skills which have impeded their progress. For all these pupils, the National Strategy offers the opportunity to catch up.

Early targeted support will help the pupils most, as it is much easier to catch problems early on than to struggle with a backlog. The *Springboard 7* materials are available to help you to provide for Year 7 pupils but it is essential to base this programme on an early and complete analysis of Year 6 attainment data. You will need to focus on the misconceptions or weaknesses the pupils have had with earlier work and build in some extra consolidation. At regular intervals, assess and review their progress and make sure that their learning of key objectives, in particular, is secure. Tell them regularly what progress they are making. Some mentoring sessions may be needed for pupils who are disaffected or whose behaviour causes concern to prepare them for whole-class work. You may be able to encourage parents to help their children in specific ways.

It is crucial that pupils in 'catch-up' groups are not withdrawn from mathematics lessons for their year group since they need to maintain the development of their mathematical skills along with their peers. More than most, they need to consolidate new learning as well as catching up on unlearned skills. Extra support is of great benefit, perhaps in tutorial time, at lunchtime clinics or after-school homework clubs, or in extra timetabled sessions. Teachers or other adults can then help particular pupils to prepare for or to consolidate their learning. For example, they could be introduced to new vocabulary in advance of a lesson, or practise skills that will be required when new work is introduced, perhaps by using suitable computer software.

The *Framework for teaching mathematics from Reception to Year 6* has a supplement of examples for Years 4, 5 and 6. Teachers who are working with lower attaining pupils in Years 7, 8 or 9 may find it useful to refer to these examples.

Pupils with difficulties in reading or writing

Reading or writing difficulties, or lack of familiarity with the English language, can slow some pupils' progress with mathematics. Teach the pupils the specific mathematical vocabulary for a unit of work and provide opportunities to refer back to this vocabulary in their own exercise books, on charts, diagrams and wall displays as well as in oral questioning. While pupils must be familiar with essential vocabulary and instructions, it helps them if you minimise written explanations on worksheets and exercises. Wherever possible, read through and discuss with them questions from textbooks.

Remember that mathematics has a strong visual element and capitalise on this wherever you can to illuminate meaning, making all illustrations directly relevant to the text. Make frequent use of visual aids such as number lines, place value charts, diagrams, mathematical shapes, measuring equipment, graphs, computer software… and games and puzzles where the rules are picked up quickly by watching a demonstration. Use games and activities with familiar rules and conventions, like Bingo or Countdown, and use them to practise different skills.

Pupils learning English as an additional language (EAL)

It is all too easy to underestimate what pupils can do mathematically simply because they are new learners of the English language. The expectation should be that they progress in their mathematical learning at the same rate as other pupils of their age. Whole-class work can provide helpful adult models of spoken English, and time for careful listening, oral exchange and supportive, shared repetition. Group work allows intensive, focused teaching input. You will probably need to direct specific instructions to EAL pupils and to speak more slowly, emphasising key words, particularly when you are describing tasks that they are to do independently. In oral work, it may help to use extra visual clues or gestures, or translation. Use picture cues on written materials and simplify the words, but not the mathematics (except where an EAL pupil also has special educational needs that warrant this).

Peer-group talk helps pupils to make sense of and apply mathematical ideas. It helps if English-language beginners can converse with other pupils or adults who speak the same home language when they are doing practical activities, playing mathematical games or working with a computer. All pupils can benefit from the experience of playing mathematical board games and puzzles from different cultures; those who know the games can show others the rules and strategies.

Aim not to ask individual pupils at the early stages of learning English to present their work orally before they are ready. Allow them time to watch and listen to those fluent in English explaining and demonstrating their methods to the class using a board or OHP. Invite them also to work through a question – they will often show capabilities that are as good if not better than their peers – but without any pressure to accompany their demonstration with an oral explanation in English before they are ready.

Emphasise the importance early on of them learning to understand, say, read and write numbers in English, signs and symbols such as percentage (%) and pi (π) and words such as ‘power’ and ‘root’. They may well be familiar with the meanings of such words in their home language. Encourage them to join in things that all pupils might do in chorus: extending a number sequence, counting along a decimal number line, chanting a multiplication table, reading an equation or inequality aloud, and so on. If you can, go through things twice with them so that they get a chance to listen and repeat. As soon as English-language beginners are reasonably confident at saying something together with others, ask them to say it again on their own. Give them plenty of time and check their understanding regularly.

Pupils with special educational needs (SEN)

Some pupils with special educational needs will also have disabilities. Some but not all may have problems with mathematics caused by difficulties of varying degrees of complexity. These problems are often, but not always, in association with literacy problems and are sometimes aggravated by missed or interrupted schooling, perhaps resulting from long-term medical conditions.

In many cases, pupils’ needs will be met through differentiation of tasks and materials. A smaller number of pupils may need access to specialist equipment and approaches, or to alternative or adapted activities. For example, there may be pupils in a class who need support in order to take part in whole-class work, such as:

- specific help with the recall of mathematical facts, to compensate for difficulties with long- or short-term memory;
- help with the interpretation of data represented in graphs, tables or charts to compensate for difficulties with visual discrimination;
- access to tactile and other specialist equipment for work on shape, space and measures, to overcome difficulties in managing visual information;
- help in interpreting or responding to oral directions, to compensate for difficulties in hearing or with auditory discrimination.

This support may be augmented by advice and further support from external specialists as described in the SEN Code of Practice or, in exceptional circumstances, with a statement of special educational need.

It is not possible in this document to give detailed advice covering every type of special educational need. As a general guide, aim to include all pupils fully in mathematics lessons so that they benefit from the oral and mental work and take part in watching and listening to other pupils demonstrating and explaining their methods and solutions. Identify relevant objectives from the teaching programmes, use suitable teaching strategies and give support so that the pupils can access lessons. Use the Framework to identify one or two suitable key objectives to incorporate into individual education plans (IEPs), tracking back to earlier stages and the primary Framework if it is appropriate to do so. Keep these objectives in mind when you plan, so that you can address special needs through simplified or modified tasks and the use of support staff to consolidate key points. Where appropriate, you could develop a more manageable 'group education plan' with common learning targets for a group of pupils who have similar difficulties.

Pupils with disabilities

Support for most pupils with physical or sensory disabilities will generally take place in the mainstream lesson as they work on the same mathematics programme as their peer group. Modifications to materials, equipment and furniture will help to meet the pupils' particular needs so that they can work alongside their peers. For example, some pupils will need to use ICT to assist them in reading or recording their work. Pupils with hearing or visual impairments may need to be appropriately positioned in a class or helped to take part in an activity through signing or support by another adult. Other adaptations that may be necessary are, for example, preparation for oral and mental work and the pace at which it is conducted, the use of signing, Braille and symbols, and the provision of materials that can be physically manipulated, including specific ICT aids and adapted measuring equipment.

Although pupils with disabilities often need time to become proficient with aids, expectations for them should remain high, with the focus on giving them maximum access and independence.

Pupils with emotional and behavioural difficulties

Many pupils with emotional and behavioural difficulties have poor literacy and numeracy skills as a result of their inability to maintain concentration and persevere with tasks. Yet if the work they are given is pitched at too low a level, they become demotivated and disaffected. The National Strategy can benefit the pupils in several ways. For example, the Framework for teaching mathematics helps to ensure high expectations for their learning. They also respond well to structured mathematics lessons where the rules, expectations and routines are well established, and the pace and level of oral work engages their interest and attention. In the main part of the lesson, plan independent

activities and written work in manageable 'chunks' so that pupils learn even when support staff are not available, and invest time in establishing routines for the transition between one activity and another.

Learning to work independently with increasing self-confidence is important for these pupils. This has to be introduced slowly, cultivated deliberately and rewarded as the level of support is reduced and eventually withdrawn. Tasks and timings are critical; plan them carefully so that you maintain pupils' motivation and interest.

Pupils with communication difficulties

Pupils with communication difficulties face particular challenges in mathematics. They need clear, effective teaching, which steadily builds their confidence and participation. Use a structured approach to the mathematical language that you use and expect the pupils to use. Some pupils with speech and language impairments have no other developmental difficulties and their mathematics lessons provide the opportunity to work alongside peers, practising and discovering strategies to overcome their difficulties. However, pupils who have autistic spectrum disorders require well-structured lessons with clear routines and predictable parts. They respond best when the language used is concise, teaching is explicit and challenges are direct and well focused. Your expectations for what these pupils will learn and do, both in the lesson overall and in each separate part or activity, need to be defined very clearly.

Pupils who are working well below national expectations for their age group

For pupils whose attainments fall significantly below age-related expectations, a much greater degree of differentiation will be necessary. You may need to refer to the teaching programmes for Key Stage 2 or Key Stage 1, modifying the ideas to set them in a context suited to 11- to 14-year-old pupils. Extra 'small steps' can be inserted, and contexts for practical work and problem solving adapted. There will then be time for consolidation without sacrificing the breadth of the teaching programmes or the principle of planning from clearly defined objectives. The *Framework for teaching mathematics from Reception to Year 6* is a useful resource for finding objectives leading progressively up to the challenges of Year 7.

Some pupils may be working at pre-level 1 for much of their secondary education. QCA intends to publish curriculum guidelines for pupils whose attainment by the age of 16 is expected to remain within the range from pre-level 1 to level 2. Challenging mathematics targets for these pupils may be found in the earliest stages of the primary Framework.

Using teaching assistants, including support for SEN and EAL

Support staff, where they are available, can help to make sure that particular pupils participate in their mathematics lessons as independently as possible. The aim is still inclusion – support is not a substitute for careful thinking about including everyone in the lesson. The success of the support will depend on good communication and working relationships between the mathematics department and the staff managing individual pupil support.

Give support staff copies of this Framework and, if possible, involve them in planning and departmental meetings. Brief them thoroughly about each lesson and their particular role in it. Make sure that they know not only what pupils are to do but also what they are to learn. Draw their attention to the key vocabulary to focus on.

During any whole-class oral work – both the starter and the teaching input in the main part of the lesson – ask support staff to position themselves close to any pupils who need special help and provide this discreetly, for example, by:

- prompting shy or reticent pupils;
- signing or translating core vocabulary or phrases;
- helping pupils to use specific individual resources, such as number cards, number lines, notebooks or small white-boards for jottings, or calculators;
- operating individualised ICT resources as indicated in a pupil's IEP.

They should also observe carefully the responses of the pupils they will be working with to inform the support they will provide.

While pupils are working on activities and written tasks in the main part of the lesson, support staff should work with identified pupils, providing help by:

- ensuring that pupils interpret instructions correctly, concentrate and behave responsibly;
- reminding pupils of teaching points made earlier in the lesson;
- questioning pupils and encouraging their participation, using questions and prompts that you have suggested;
- using and helping pupils to use number lines and squares, calculators and other ICT devices, visual or practical aids…;
- looking for and noting any common difficulties that pupils have, or mistakes that they make, so that you can address these in the plenary and in future lessons;
- helping to recognise pupils' successes so that they can be moved on.

Using the Framework in special schools

Many of the Framework's principles are applicable to special schools, such as planning from clear teaching objectives, an emphasis on oral and mental work, visual interest, involvement and interaction, and keeping pupils working together as far as possible. Special schools are encouraged to adopt the Framework but should also adapt the guidance to suit their particular circumstances. For example, the notion of 'whole-class teaching' can be modified, as it is different when there are 10 to 14 pupils in a class, and the teacher is at times supported by other staff. There may be times when all the pupils are taught together for their mathematics lesson, just as in a mainstream school. At other times two 'whole-class lessons' may be taking place in the same room, with the class teacher teaching one half of the class, and a support assistant working with the other half.

In some special schools all or nearly all of the pupils in a class will have learning difficulties that extend to mathematics. For these pupils, the routine of the mathematics lesson is best built up over a period of weeks. Aim first to establish regular oral and mental work. Then introduce routines for main part of the lesson and the plenary, concentrating on a strong oral or other communication technique which promotes interaction, combined with a good range of practical activities.

Assessment and target setting

As pupils come to terms with their new school, it is essential to continue to build their mathematical skills. Year 7 teachers need to know what their pupils can already do. Many pupils now leave Year 6 with personal targets, records and a history of intervention. This body of information can help secondary teachers to make a quick start on work that is well matched to pupils' capabilities. The 'clean-sheet' approach is too slow, and allows pupils to coast or to fall back when they need to be challenged. As a minimum, teachers of Year 7 classes should survey the information available to them about the mathematical attainment of incoming pupils to help plan in advance the work of the first term, and then review individual records more closely when staff have had three or four weeks of experience with individual pupils. This alerts them to unexpected changes in performance which need to be resolved and enables them to adjust teaching expectations accordingly.

Priorities for each new cohort can be derived from Key Stage 2 levels and raw scores and the qualitative information provided by work sampling and other monitoring in the early part of Year 7. This helps the department to translate wider ambitions such as 'improve number skills' first into numerical targets such as 'increase by 5% the proportion of pupils achieving Level 4 in number and algebra by the end of Year 7', then into specific curricular targets such as 'all pupils will recognise the equivalence of simple fractions, decimals and percentages' or 'all pupils will use the order of operations, and know that algebraic operations follow the same conventions and order as arithmetic operations'.

Assessment, recording and reporting are important elements of teaching but they have to be manageable if the information they yield is to be useful. The best assessment has an immediate impact on teaching, because it alerts you to the needs of pupils who are either out of step or exceeding expectations. Assessment should help to maintain the pace of learning for all pupils by informing teaching plans, in a continuous cycle of planning, teaching and assessment.

It is useful to consider assessment at three connected levels: short-term, medium-term and long-term.

Short-term assessments

Short-term assessments are an informal part of every lesson. Their purposes are to:

- check that pupils are developing mental skills: for example, that they can recall mathematical facts, estimate, calculate mentally, and use visual imagery;
- check that pupils have grasped the main teaching points in a particular lesson or unit of work, whether they have any misunderstandings that you need to put right, and whether they are ready to move on;
- give you information which will help you to adjust day-to-day lesson plans and brief any support staff about which pupils to assist, and how to assist them.

For these short-term assessments what you assess will be matched closely to your teaching objectives. There are three main ways to make them:

- **During every lesson** you absorb and react to pupils' responses, see whether they are confident or hesitant with new work, decide whether they need extension work or more help, and offer immediate support. Where you notice any difficulties or

misunderstandings, you adjust your lesson and address them straight away, if necessary continuing in the next lesson or two. In this way, pupils can keep up with the pace of work and do not fall behind. In the plenary, you can acknowledge individual and class achievement and effort and remind pupils about their targets. The plenary is also a good time to firm up short-term assessments by asking probing questions to judge how well pupils have understood new work and to check again for any misconceptions.

- **At intervals** you will supplement your day-to-day observations. For example, a homework task or an occasional short informal test can give you useful information on who has learned what and who needs extra support.
- **Marking** of pupils' work will be followed immediately by discussion with the class to give pupils feedback on their performance and what they need to do to improve. At the same time you can make sure that any errors are put right and the merits of different methods or approaches discussed.

Short-term assessments don't need to be recorded, since they are for immediate action and attention. Some teachers note briefly when a pupil surprises them, perhaps with his or her knowledge, or with something that is unexpectedly difficult. These informal, personal recordings can help to clarify patterns in performance over time or responses to specific teaching or support.

Medium-term assessments

Medium-term assessments should gather new information, not just confirm what you already know. They are mainly to:

- review and record pupils' progress over the previous half-term or term in relation to key objectives, what they know and can do, whether they can apply their skills in a new context, and whether they still have any difficulties;
- identify pupils' progress against specific individual targets, including those in IEPs, so you can give pupils feedback and set new targets;
- help you to plan work over the next half term or so;
- give you information to feed into end-of-term or end-of-year assessments.

Most pupils should be living up to expectations for their class and you will be familiar with their progress and learning from your short-term assessments. Medium-term assessments should centre on the most important aspects of the mathematics you have taught and help you to identify pupils' particular strengths and weaknesses. They should relate particularly to the **key objectives** that you have focused on in the half-term's work (for details of key objectives, see Section 2). These objectives are central to all pupils' progress in relation to the National Curriculum level descriptions, and hence to their performance in tests and teacher assessments.

Medium-term assessments are best timed to influence planning. At intervals, perhaps at the end of a half-termly block of several units of work, there is an obvious opportunity to assess how well pupils have done against the relevant key objectives, and to set targets for the future. This may mean, for example, a review of work completed in the units, a substantial assignment, a test, or an assessment of oral and mental skills. Choose tasks that pupils can tackle independently so that you can concentrate on the pupils you are unsure about. The results need not be elaborate: if the units have gone well, it may just be a matter of identifying which pupils need extra feedback or consolidation, and setting new targets for the whole group. It may, on the other hand, suggest certain targets for

particular groups and individuals. Some of these targets will take pupils on to new objectives, and others will ensure that insecure learning is consolidated. The principle is to mobilise assessments quickly into the setting of relevant and realistic targets.

Marking

Marking of tests or assessment tasks should usually be followed by giving pupils feedback on what they have achieved and how to improve their work. You will probably want pupils to make corrections, so constructive written comments are more helpful than grades and ticks and crosses. Marking, feedback, corrections and rectifications of difficulties are best done immediately so that pupils can still remember how they approached the task and so that you can modify your teaching plans if you need to.

Recording

Pupils' progress towards the key objectives needs to be recorded. Since there are relatively few key objectives for each year, records are not too onerous to maintain, and updating them every six weeks or so is sufficient. The easiest system is a **class record of the key objectives**, for example:

Key objectives: Year 8														
Add, subtract, multiply and divide integers.														
Use the equivalence of fractions, decimals and percentages to compare proportions; calculate percentages and find the outcome of a given percentage increase or decrease.														
Divide a quantity into two or more parts in a given ratio; use the unitary method to solve simple word problems involving ratio and direct proportion.														
Use standard column procedures for multiplication and division of integers and decimals, including by decimals such as 0.6 or 0.06; understand where to position the decimal point by considering equivalent calculations.														
Simplify or transform linear expressions by collecting like terms; multiply a single term over a bracket.														
Substitute integers into simple formulae.														
Plot the graphs of linear functions, where y is given explicitly in terms of x; recognise that equations of the form $y = mx + c$ correspond to straight-line graphs.														
Identify alternate and corresponding angles; understand a proof that the sum of the angles of a triangle is 180° and of a quadrilateral is 360°.														
Enlarge 2-D shapes, given a centre of enlargement and a positive whole-number scale factor.														
Use straight edge and compasses to do standard constructions.														
Deduce and use formulae for the area of a triangle and parallelogram, and the volume of a cuboid; calculate volumes and surface areas of cuboids.														
Construct, on paper and using ICT, a range of graphs and charts; identify which are most useful in the context of a problem.														
Find and record all possible mutually exclusive outcomes for single events and two successive events in a systematic way.														
Identify the necessary information to solve a problem; represent problems and interpret solutions in algebraic, geometric or graphical form.														
Use logical argument to establish the truth of a statement.														
Other														

Your class record can be dated when you feel confident that a pupil has achieved a key objective. This is a sufficient record of most pupils' progress but you will probably need some supplementary notes for the few individuals whose progress towards the key objectives differs markedly from the majority of their peers. A class record of this kind can be a useful aide-mémoire for parents' evenings or when you are writing annual reports.

Individual targets for pupils

A discussion with pupils during the course of each half term to set them personal targets helps them to achieve the key objectives over the medium term. You may want to arrange your discussion with some pupils on an individual basis – for example, pupils with special needs whose IEPs need updating, or pupils who would benefit from a degree of privacy – but for most of them you can organise the discussion in small groups as part of an ordinary mathematics lesson. Ask pupils to suggest two or three improvements to work on over the next term. You could also offer pupils some practical advice on the steps they might take to achieve their targets, and give them an occasional opportunity to work on the targets as part of one or more homework tasks. It is helpful if some monitoring of progress towards individual pupils' mathematics targets can take place in tutor-group time as part of a whole-school approach to target setting.

Individual targets will usually be linked to the key objectives that you will focus your teaching on over the next few weeks. They may be very specific: for example, to become proficient at adding and subtracting a pair of fractions. For some pupils a target may need to be broken down into stages: for example, to learn to add a pair of unit fractions. For others, it may be appropriate to choose a target linked to the key objectives for the year group below or above. Whatever the targets, they need to be straightforward and not too many at one time, so that pupils understand them.

You can note the targets you set for individual pupils by highlighting the particular boxes on a class record of key objectives. Exceptionally, there may be some pupils with special needs whose personal targets need to be recorded in your supplementary notes or IEP.

Long-term assessments

At particular times, and especially towards the end of the school year, you will assess and review pupils' progress and attainment against school and national targets, drawing on your class record of key objectives and supplementary notes.

Long-term assessments are also important in Years 7 and 8, not just at the end of Year 9. Their purposes are to:

- assess pupils' work against the key objectives for the year;
- at the end of Year 9, assess pupils' work against national standards;
- give you supplementary information about individual pupils' attainment and progress so that you can report to parents and, if appropriate, the next teacher;
- help the school to set targets for mathematics for future years;
- allow the headteacher to brief governors and others on progress and attainment in Key Stage 3, including progress towards school, LEA and national targets.

Long-term assessments include end-of-year tests or examinations, and teacher assessments.

- The compulsory National Curriculum mathematics tests for Year 9 can be supplemented by the optional tests for Years 7 and 8 provided by QCA. The test scores will help you to monitor whether pupils individually and collectively are attaining at, below or above 'national expectations', and how their attainment compares with their previous attainment. Results expressed as National Curriculum levels help you to judge overall standards and progress towards school, LEA and national targets. Each year QCA publishes a *Standards Report* for Key Stage 3, analysing pupils' performance on the Year 9 tests. These reports also help to identify particular weaknesses which you may need to tackle in your next phase of teaching.

- You will also make a teacher assessment to sum up your judgement of pupils' attainment. For Year 9, this statutory end-of-year assessment will be against the National Curriculum level descriptions. The cumulative picture that you carry in your head of the progress of each pupil can be extended and secured by looking through samples of pupils' work. At the same time, you can update and complete class records of key objectives, and any supplementary notes you have made.

Before end-of-key-stage teacher assessments are made, it is helpful if all staff teaching mathematics in Key Stage 3 examine together a sample of pupils' work from each class. A moderation exercise helps to make sure that judgements against the National Curriculum level descriptions are consistent through the department. The exercise could also be extended to Year 7 and Year 8 to establish clear expectations against the key objectives.

Planning

Good planning is worthwhile. The time involved is an investment for future years and reduces the demands of paperwork in the long term. Experience in primary schools of planning from the Framework suggests that it becomes quicker and easier with familiarity. Plans that are well constructed and informative do not have to be written in full prose or elegantly typed, though they do need to be accessible to others. Their prime purpose is to specify coverage, ensure good progression and so improve teaching.

It is helpful to consider three levels of planning which together form a scheme of work.

Long-term plans

The Framework provides yearly teaching programmes – sets of objectives of what to teach over the course of each year, based on the National Curriculum programmes of study for the key stage.

Medium-term plans

Medium-term plans show the titles of units of work or main topics that pupils will be taught over a half-term or term. Each unit covers a block of several lessons. There are four basic requirements for each year group:

- for each unit, the number of hours or lessons is specified;
- the objectives to be addressed in each unit are identified, with adjusted objectives as appropriate for higher and lower attainers;
- for the majority of pupils in each year, the units between them cover all the objectives for the year;
- taken as a whole, the units provide breadth and balance across the National Curriculum programmes of study.

The best medium-term plans provide opportunities to revisit topics and to make connections between different aspects of mathematics. They also build in time for regular assessment and review.

Short-term or lesson plans

Short-term or lesson plans are the teaching notes for a block of lessons. They show how a unit of work will unfold to meet the intended objectives. They indicate the objectives for each lesson or group of lessons that form part of the unit, outline starter activities, show how work will be developed in the main part of the lessons through teaching input and pupil activities, indicate how lessons will be rounded off and suggest what homework will be set. They may include references to relevant resources, such as textbooks and ICT applications. Each unit will consist of several lessons: for example, an 8-hour unit may be planned in a sequence of 3, 2 and 3 lessons. A unit may, of course, include a plan for a single key lesson, perhaps an introductory lesson based on a problem, or a lesson focusing on an open-ended investigation suitable for pupils with a wide range of attainment.

There could be up to three versions of a particular unit of work for a specific year group, differentiated for higher, average and lower attainers, but with some overlap.

The main requirement of a short-term plan is that it makes clear how the objectives for the relevant unit will be taught. It should be possible to trace an objective from the yearly teaching programme in the Framework through its location in the medium-term plan, down to when and how it will be taught in one or more lessons.

The yearly teaching programmes

The Framework contains a set of double-page **yearly teaching programmes** summarising teaching objectives for mathematics for each of Years 7 to 9 (see Section 3). The **key objectives** are highlighted in bold in the teaching programmes and are also listed separately (see Section 2).

The teaching programmes continue and extend the progression and expectations in the primary Framework, so that work in each year corresponds to these levels:

Year 5	revision of level 3, but mainly level 4
Year 6	consolidation of level 4, and start on level 5
Year 7	revision of level 4, but mainly level 5
Year 8	consolidation of level 5, and start on level 6
Year 9	revision of level 5, but mainly level 6 objectives for able pupils at level 7, with some at level 8

The yearly teaching programmes for Years 7 to 9 are organised in six strands, linked to the National Curriculum programmes of study for mathematics in Key Stage 3. The six strands and the topics that they cover are:

Using and applying mathematics to solve problems

- problem solving and applications in a variety of contexts to develop reasoning, thinking and communication skills

Numbers and the number system

- place value, ordering and rounding
- integers, powers and roots
- fractions, decimals, percentages, ratio and proportion

Calculations

- number operations and the relationships between them
- mental methods and rapid recall of number facts
- written methods
- calculator methods
- checking results

Algebra

- equations, formulae and identities
- sequences, functions and graphs

Shape, space and measures

- geometrical reasoning: lines, angles and shapes
- transformations
- coordinates
- construction and loci
- measures and mensuration

Handling data

- specifying a problem, planning and collecting data
- processing and representing data
- interpreting and discussing results
- probability

The supplement of examples

After the teaching programmes is a **supplement of examples** (Section 4) to illustrate what pupils should know and be able to do by the end of the year. The examples are a selection, not a full set, and are not intended to be taught as a 'scheme of work' or used on a series of worksheets. Their main purpose is to help you first to interpret the level of the work and then to plan, teach and assess it so that there is steady progression throughout the year, and from one year to the next. They could also be used to help you formulate pupil target statements. Page references in the teaching programmes identify the relevant pages of examples in the supplement.

Objectives from the teaching programmes are described in the left-hand column, and illustrated for Year 7, Year 8 and Year 9 in the other three columns across a double-page spread. Examples illustrating the objectives for able pupils in Year 9 are included in the Year 9 column in blue italic. Cross-references to other topics are included to help establish the connections between them.

Making connections

The six strands are described separately in the yearly teaching programmes but there are many links between them. Mathematics is not a group of isolated topics or learning objectives but an interconnected web of ideas, and the connections may not be at all obvious to pupils. Providing different examples and activities and expecting pupils to make the links is not enough; pupils need to be shown them and reminded about work in earlier lessons. You could do this in the main teaching activity through explanations, demonstrations and illustrations but you could also use plenary sessions to draw pupils' attention to links with past or future learning.

Good planning ensures that mathematical ideas are presented in an interrelated way, not in isolation from each other. Awareness of the connections helps pupils to make sense of the subject, avoid misconceptions, and retain what they learn. So when you plan:

- As far as possible, present each topic as a whole, rather than as a fragmented progression of small steps: for example, show pupils that the place value system encompasses both whole and decimal numbers of any size, and that decimals and percentages are particular forms of fractions.
- Bring together related ideas across strands: for example, link work on metric measures to the decimal place value system, or link ratio and proportion in number to rates of change in algebra, to enlargement and similarity in geometry, and to proportional thinking in statistics and probability.
- Help pupils to appreciate that important mathematical ideas permeate different aspects of the subject: for example, the concepts of *inverse* and *order* link the four number operations, are the key to the transformation of algebraic expressions, and are central to the geometrical transformations of reflection, rotation and translation.
- Use opportunities for generalisation, proof and problem solving to help pupils to appreciate mathematics as a unified subject: for example, proof and mathematical argument involve chains of reasoning which link ideas together, so to prove that the sum of any three consecutive numbers will always be a multiple of 3 might involve forming algebraic expressions, using the distributive law to factorise an expression and recognising the link between factors and multiples.

Medium-term planning charts

On the next three pages there are some examples of medium-term **planning charts**, suggesting how units of work might be constructed and timed for Years 7, 8 and 9. Other groupings of units could be equally suitable – there is no one solution that is likely to be the best for all schools and every timetable. You could adapt the sample charts to reflect the number of lessons you have, and the lesson structure you adopt, or design your own.

The assumption is that each yearly programme can be taught in 35 weeks, about 105 hours of teaching time. Time for assessing and reviewing pupils' progress needs to be built into this time. The recommended number of teaching hours can be converted to an appropriate number of lessons for your timetable. For example, a unit of 6 hours could be adapted to five 70-minute periods or seven 50-minute periods. Time beyond the 105 hours could be used for extended problem solving, extra reinforcement, or revision.

Units in the example charts are organised to ensure that there is an appropriate balance of work in each term and across the year. The overall proportions of teaching time are about 55–60% for number and algebra, 25% for shape, space and measures, and 15–20% for handling data, in line with QCA's recommendations for the National Curriculum. You could alter this balance for particular reasons: for example, to allocate a little less time to one unit and a little more to another, to take account of your pupils' progress and their particular strengths and difficulties.

In Year 7, the units in the sample charts are shorter. Topics are revisited at least once to consolidate and develop the ideas that have been introduced. By Year 9, the units are longer and incorporate more topics to allow a greater focus on the connections to be made between different aspects of mathematics.

One way of using planning charts, either the examples on pages 48–50 or your own, is to identify and note the particular objectives that will be the main focus in each unit, making sure that you include some 'using and applying' or 'thinking skills' objectives alongside the main content objectives.

Start with the objectives for middle attainers, drawing from the appropriate yearly teaching programme. Aim to include each objective from this programme at least once by the end of the school year.

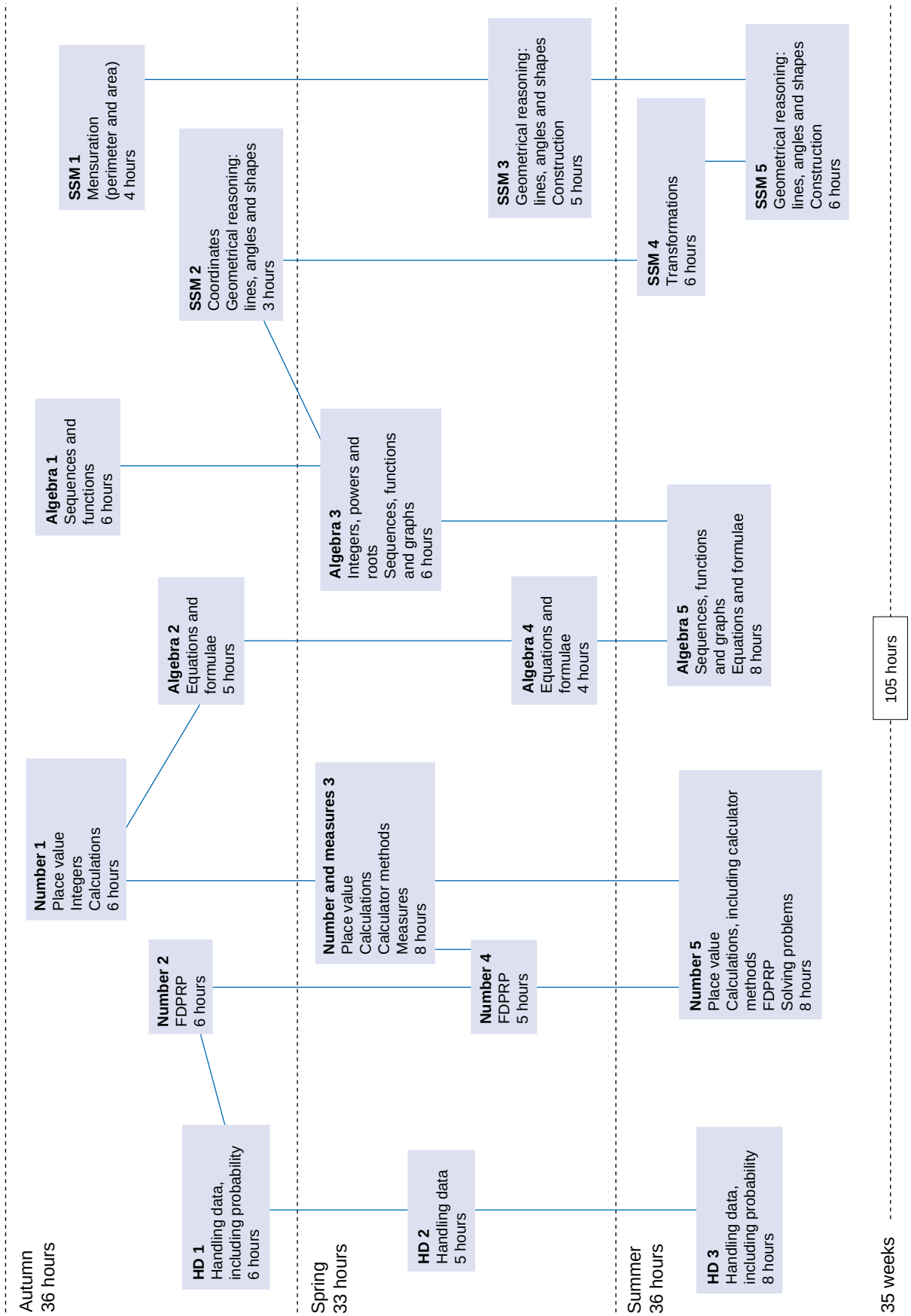
Next, select the objectives for more able pupils who are secure with the mathematics for their age group, drawing these from the corresponding topics in the next yearly teaching programme. For those Year 9 pupils who are likely to achieve level 8 in the Key Stage 3 tests, you may need to look beyond the objectives for able pupils in Year 9 and draw on topics from the higher programmes of study for Key Stage 4.

For pupils who need to consolidate work from earlier years, draw appropriate objectives from the corresponding topics in earlier yearly teaching programmes. For example, pupils in Year 7 who have yet to achieve level 4 may need to follow a programme of work based largely on objectives in the Year 5 and Year 6 yearly teaching programmes, leaving the higher level work – particularly algebra – until later in Key Stage 3. They may also need extra support and teaching time to make sure that they move on to Year 7 work as soon as possible (see 'Pupils who need to catch up', page 34). The *Springboard 7* materials for the autumn and spring terms are designed to support this approach.

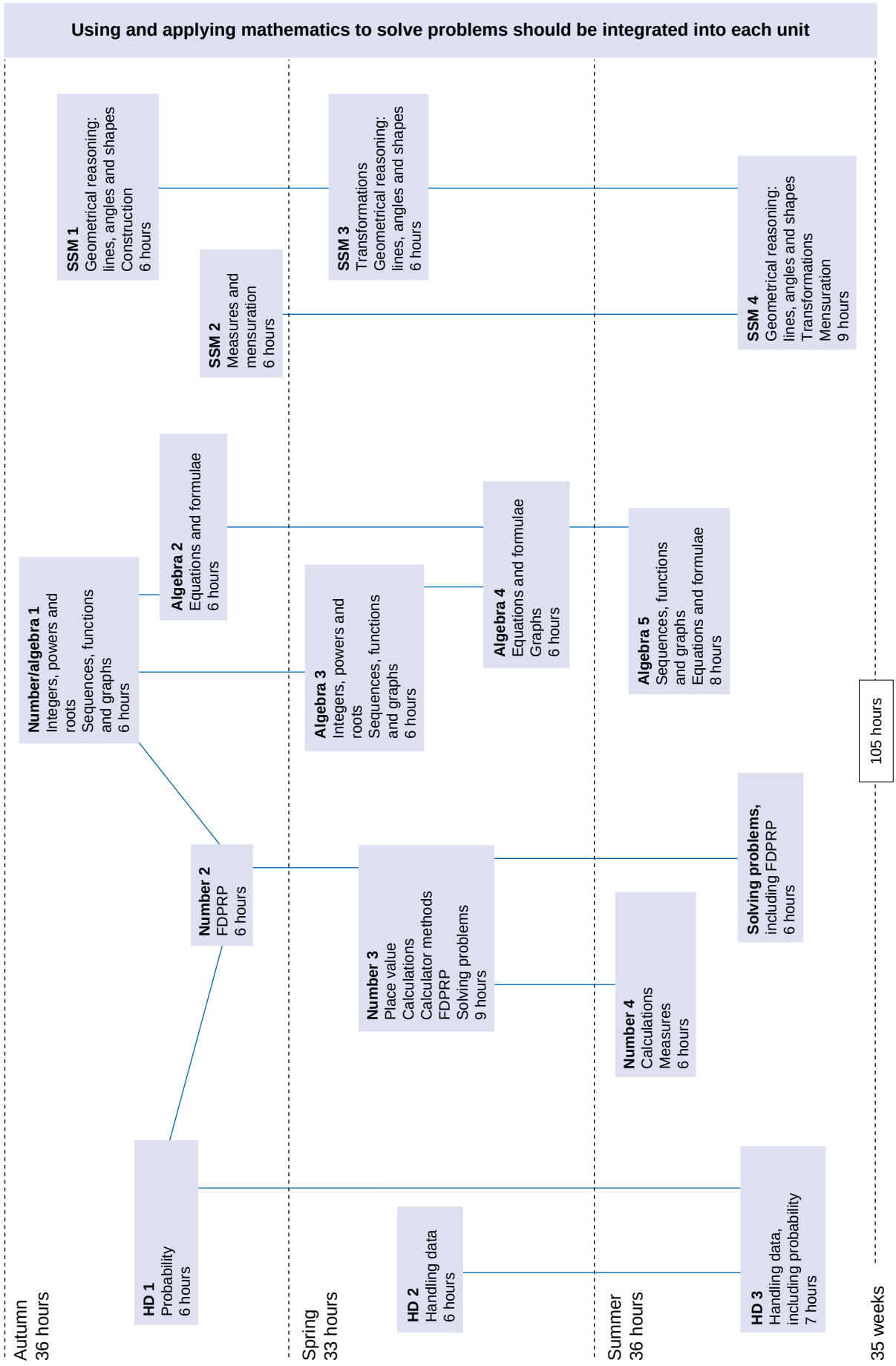
Planning charts like these, with the accompanying objectives for each unit, are equivalent to the primary planning templates. When the associated short-term or lesson plans (i.e. the teaching notes for a block of lessons) are added to them, they are equivalent to a scheme of work. Schools do not need a separate or extra document to fulfil this purpose.

Example planning chart and possible units for Year 7

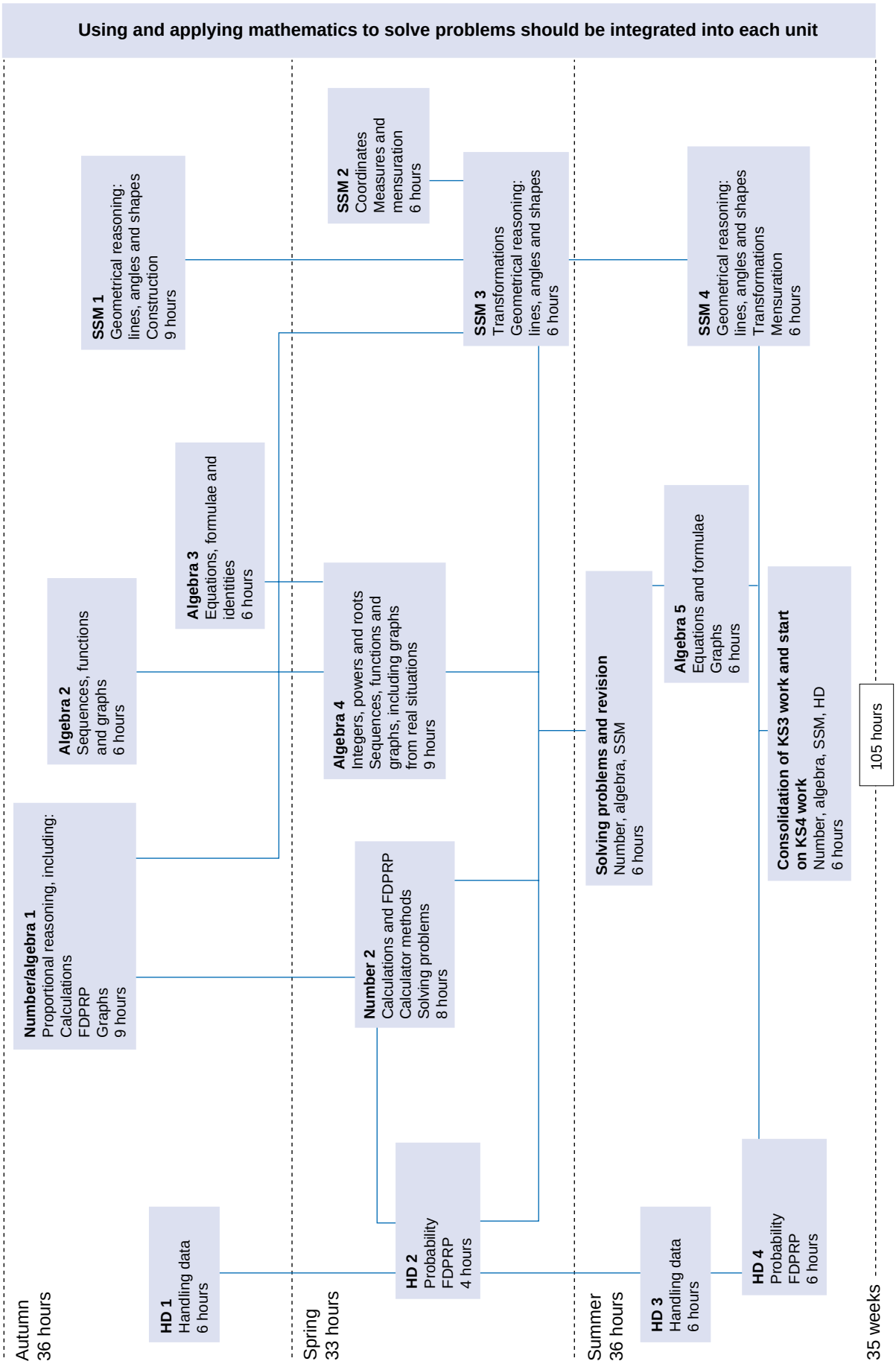
Using and applying mathematics to solve problems should be integrated into each unit



Example planning chart and possible units for Year 8



Example planning chart and possible units for Year 9



The approach to planning

The staffing and organisation of mathematics in Key Stage 3 is different from that in Key Stages 1 and 2. Most teachers:

- are subject specialists who teach only or mainly mathematics;
- teach up to five or more year groups and so have a good grasp of progression in the subject;
- teach classes across the ability range and recognise pupils' differing needs.

Each year group may be taught by a significant proportion of teachers in the mathematics department. Even where pupils are organised in ability sets, there is much overlap across different classes of what is planned and taught.

These differences affect the approach to planning to a certain extent. Experienced teachers will generally have a good grasp of subject content and progression, but consistency across the department and economy of effort are helped by a collaborative approach to planning each unit. Consistency helps to ensure higher standards and aids movement of pupils between sets when this is necessary.

Some starting points for discussion will stem from the audit of mathematics. For example:

- Are objectives for what we will teach clearly defined for each lesson or group of lessons?
- How well does our scheme of work cope with the range of attainment across a year group? Is there consistency in our expectations for pupils and in our approaches to planning and teaching?
- How can we best ensure an appropriate pace of learning for each class?
- What is the most effective way to plan collaboratively?
- What should be expected of individual teachers in terms of lesson-to-lesson planning?

Reviewing your current scheme of work and medium-term planning

Although this Framework implies new emphases to planning and teaching mathematics, it is possible that some existing schemes of work will be easily adapted. Most important is the need to plan around specific objectives in the Framework: simply touching on them is not enough. So the job of aligning existing schemes of work to the Framework is more than a tick-list activity.

A successful review of a scheme of work depends on strong and enthusiastic leadership. It involves mathematics staff regularly reviewing and updating units in the current scheme of work, with detailed work delegated to subgroups. Joint working helps to spread good practice and ensure consistency in the approach to teaching across the department. The outcome of the review can be in the form of revisions or annotations to relevant units in your current scheme of work, or a redraft of the units where you feel this is necessary. After trying out revisions, feedback from teachers will identify any adjustments needed, and help the scheme of work to evolve.

Key tasks

The first aim is for all mathematics teachers to become familiar with the expectations of the Framework. Initial tasks might include these.

- Compare the objectives in the yearly teaching programmes for Years 7, 8 and 9 with what your school does now. Ask: How do they match up? Do we need to make any adjustments? If so, what strand of our scheme of work shall we review first?

- Choose a particular topic for analysis, such as equations and formulae, and look at the supplement of examples for Years 7, 8 and 9. Ask: Do the examples match our expectations? How might they help with our planning?

The next task is to cluster together groups of teaching objectives into units of work for the main teaching activities in Year 7, drawing on objectives for Year 6 or Year 8 where this is appropriate for your pupils. The sample charts on pages 48–50 are examples showing how time might be distributed in each year between units or different mathematical themes. They also provide an overview of the year's work.

For example, in the unit Algebra 3 in the sample chart for Year 7, the main teaching objectives, which include some 'using and applying' objectives, might be as follows:

ALGEBRA 3: YEAR 7 **6 HOURS**

Integers, powers and roots
Sequences, functions and graphs

SIMPLIFICATIONS from the Y6 teaching programme	CORE from the Y7 teaching programme	EXTENSIONS from the Y8 teaching programme
• Recognise multiples up to 10×10 and squares to 12×12; know and apply simple tests of divisibility.	• Recognise and use multiples, factors (divisors), common factor, and primes less than 100; use simple tests of divisibility. • Recognise and use the first few triangular numbers, squares to 12×12, corresponding roots.	• Find the prime factor decomposition of a number. • Use squares, positive and negative square roots.
• Use a calculator to square numbers.	• Use the square root key on a calculator.	• Use the function keys on a calculator for sign change, powers and roots.
• Recognise and extend number sequences.	• Generate terms of a simple sequence, given a rule; generate sequences from practical contexts.	• Generate terms of a linear sequence using term-to-term and position-to-term definitions, on paper and using a spreadsheet or graphical calculator. • Begin to use linear expressions to describe the nth term of an arithmetic sequence.
	• Express simple functions in words, then using symbols; represent them in mappings.	• Express simple functions in symbols; represent mappings expressed algebraically.
• Read and plot coordinates in the first quadrant. • Represent and interpret data in a graph (e.g. of a multiplication table).	• Generate coordinate pairs that satisfy a simple linear rule; plot graphs of the form $y = kx$, where y is given explicitly in terms of x, on paper and using ICT.	• Generate coordinate points in all four quadrants and plot the graphs of linear functions; recognise that equations of the form $y = mx + c$ correspond to straight-line graphs.
• Solve mathematical problems, explaining patterns and relationships in words.	• Solve mathematical problems in a range of contexts (number, algebra). • Identify the necessary information; represent problems mathematically making correct use of symbols, words, diagrams, tables and graphs.	• Solve more complex problems by breaking them into smaller steps. • Represent problems in algebraic or graphical form, using correct notation.

Incorporating using and applying mathematics

All pupils need to have:

- direct teaching which develops particular knowledge and skills;
- frequent opportunities to explain, reason and justify their mathematical thinking;
- opportunities to solve problems, some of which may have extended solutions or lead to other problems, and where the focus is on the problem rather than on a particular range of techniques.

Every lesson is likely to offer some opportunity for pupils to explain and justify their mathematical thinking. In addition, every unit needs to incorporate some problem solving, with reasoning and proof, though not necessarily in every lesson. So for part of every unit, either in parts of lessons, or in one or more whole lessons, it makes sense to focus on problem solving of an open or investigative nature, basing this on suitable selected objectives drawn from 'using and applying mathematics'. Overall, this focused teaching time will probably amount to 15–20% of lesson time, or from 5 to 7 hours each term. Over a year, problem solving and investigative work should involve number and algebra, shape, space and measures, and handling data.

Some schools have devoted one lesson in each 6-hour unit to 'using and applying mathematics' and 'thinking skills'. These schools have reported spin-offs in other lessons, when pupils apply the problem-solving skills that have been drawn to their attention in the focused teaching.

Planning oral and mental starters

Suitable objectives for oral and mental starters to develop pupils' mental agility and their visualisation, thinking and communication skills will also need to be identified. Some of the objectives will link directly to the main teaching objectives of the relevant units, while others will have the purpose of rehearsing skills to keep them sharp.

Oral and mental work can be planned first in half-term blocks to ensure progression across the year. You can then decide which objectives to focus on in each unit of work. For example, in the first half of the spring term in Year 7 you might choose these objectives to work on:

ORAL AND MENTAL OBJECTIVES: Year 7, spring term, first half

- Add and subtract a series of small whole numbers, e.g. 5 + 7 – 4, 3 + 4 + 5 + 6.
- Continue an ascending sequence, e.g. start at 3, count on in steps of 7.
- Add / subtract pairs of numbers such as 7.6 ± 3.8, 760 ± 380.

- Recall multiplication and division facts to 10×10.
- Find remainders after division of two-digit numbers by 2, 3, 4, 5, 10.
- Recall pairs of factors and primes (less than 50).

- Multiply and divide decimals by 10, 100.
- Multiply and divide a two-digit number by 20, 30, 40…, using factors.

- Double and halve simple fractions, e.g. three fifths.
- Find fractions equivalent to halves, quarters, thirds, tenths.
- Recall fraction, decimal and percentage equivalents (halves, quarters, thirds, tenths).
- Find 75%, 50%, 25% and multiples of 10% of quantities.

- Visualise, describe and sketch 3-D shapes; describe their properties.
- Estimate and order acute and obtuse angles.
- Use metric units and units of time for mental calculation.

- Solve word problems mentally (two steps).
- Make and justify estimates and approximations of calculations.
- Discuss and interpret graphs.

Overall balance of the units

You will, of course, need to determine the number, order and length of your teaching units. In some cases, it will be clear when one topic should precede another. In other cases, there is no essential order and the main criterion for ordering units is how best to build links and connections into your teaching.

The overall aim is to achieve a balance between revisiting and developing topics without too long a time gap between, and ensuring that teaching is not too fragmented.

The sample charts on pages 48–50 were developed for each year group with these criteria in mind. There should be:

- breadth and balance across each term, each year and the whole key stage, to ensure appropriate coverage of the Key Stage 3 programme of study;
- consistency for all mathematics classes in a year group in the timing and organisation of the units so that, where sets are organised, joint planning of work by teachers and movement of pupils between sets are readily achieved;
- opportunities to make connections between different topics, so that objectives from different strands can be grouped together;
- regular opportunities to apply what has been learned in problem-solving contexts;
- opportunities to revisit topics regularly, through:
  - providing shorter, focused units of 3 to 6 hours in Year 7, in order to return to topics relatively frequently for consolidation and development of ideas and to encourage variety and pace;
  - providing longer, more sustained units of work in Year 9, bringing topics together to ensure revisiting and to emphasise links within and across strands of mathematics;
  - providing for all year groups half-termly sets of objectives for oral and mental starters, so that skills, once taught, can be rehearsed and kept sharp.

From medium-term to short-term lesson plans

At an early stage of discussion of a set of units across a year, it helps to identify some of the approaches to teaching and pupil activities that are to be used to ensure that the objectives of the units can be met. This core planning task raises a number of questions.

- What mathematics have these pupils been taught before? How will this unit review and build on what pupils already know, understand and can do? What are the key teaching points that we need to stress?
- What activities are most suitable in the main part of the lesson? Are the activities short or long? How many will we need? Is there suitable variety in the activities, including problem solving and applications? What resources might we use to demonstrate and model ideas and support the activities, including ICT?
- How will we develop and extend the teaching input and pupil activities through the unit? Will pupils experience an appropriate variety of teaching and learning opportunities across the whole block of lessons?
- What difficulties or misconceptions might pupils have? How can these be avoided and resolved? How can we use plenary sessions to probe misconceptions?
- How can we help pupils establish links between topics? For example, could we use earlier examples and common vocabulary to illustrate and demonstrate connections?
- Are there opportunities in other subjects to introduce or reinforce the ideas?
- What tasks will be suitable for homework?
- How will we use oral and mental starters to keep pupils' skills 'on the boil', to develop skills learned in previous units and to prepare for the current units?

One further aspect to consider is differentiation to meet the needs of different pupils, taking into account your organisation of teaching groups. No class, whether mixed ability or an ability set, will have all pupils operating at the same level all the time, so differentiation will need to be indicated in the main set of notes, usually at no more than three levels of difficulty. Where classes are set for mathematics, separate notes for

different teaching groups may be needed. For mixed ability classes some further considerations are:

- How can we adapt activities to make sure that all pupils make good progress and are suitably challenged?
- Do we need to provide smaller steps, guided tasks and supporting structures to enable pupils who are working below age-related expectations to play a full part?
- Do we need to add challenge by increasing requirements for higher attaining pupils?

As an outcome of all these discussions, a unit should provide guidance on sets of two or three lessons and cover as a minimum:

- teaching time for the lessons, and homework time;
- objectives and activities for the oral and mental starters;
- objectives to be addressed in the main teaching activities, with adjusted objectives as appropriate for higher and lower attainers;
- key teaching points and activities for lessons, matched to the objectives and with appropriate references to the supplement of examples, with suggestions for how the activities can be developed, including extensions or simplifications;
- the resources needed, including ICT, with references to any departmental files and relevant parts of textbooks;
- ideas to be drawn out in plenary sessions, including some key questions, and possible homework tasks;
- key mathematical terms and notation.

As a unit is refined and developed further it might also include:

- potential difficulties or misconceptions that pupils may have, and suggestions to pre-empt or rectify them;
- how best to deploy any available support staff;
- assessment strategies and what adjustments to future plans might be needed as a result;
- connections with other mathematical topics and other subjects;
- an occasional detailed plan for a key lesson, on a critical aspect of the topic, a particular teaching method, or the use of a particular resource such as ICT.

Individual lesson planning

Good teaching notes, for an established unit of work, should give all teachers a helpful overview and guidance on what to teach. Experienced and successful teachers with a good sense of pacing should be able to teach a block of lessons directly from the notes, taking account of pupils' progress and response to the work, and making use of informal and formal assessment information. As teachers think through what is needed they may need little more than brief personal notes to identify how each lesson will unfold. There may, of course, be occasions when more detailed lesson plans are required for each lesson, for example, for Ofsted inspections or for demonstration lessons.

In addition, all departments should have systems for supporting colleagues who may benefit from using more detailed plans for each individual lesson. These include newly qualified teachers, teachers new to the school or recently returned to teaching, students in training, non-specialist teachers and those who teach mathematics only occasionally. The teachers concerned should feel that what they record is of direct help to them and to anyone who is supporting them, such as a classroom assistant, or an experienced teacher with whom they discuss their planning from time to time.

Where to begin

The Framework is a guide to what to teach to each class. However, there may be some secondary schools where, at present, relatively few pupils attain level 5 or above at the end of Key Stage 3. Pupils may lack a secure understanding of some of the work they have been taught earlier. To begin with, these schools should look carefully at the programmes for Year 5 and Year 6 and draw suitable teaching objectives from them when they are planning work for Year 7, making corresponding adjustments for Years 8 and 9.

Where a school sets pupils for mathematics lessons, teachers of higher sets may well base their pupils' work on a programme for an older age group, while teachers of lower sets may need to work mainly from objectives in the teaching programmes for a younger age group. For example, after the first few weeks of the autumn term, a Year 7 class in a selective school, or a top set in a comprehensive school, is likely to follow a programme based largely on the Year 8 objectives.

Decisions such as these will need to be reviewed before the start of the next school year to allow for improving standards over time.